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FINANCIAL MARKETS IN A DEVELOPING ECONOMY: LOANS, PROBABILITY OF DEFAULT, AND GROWTH

by José Aníbal Insfrán P.

This paper presents a model for the market of small loans with two sectors,[1] and its influence on economic growth through the default probability. The model for loans is in the line of the one developed by Waller and Lewarne (1994), using more than one type of loan, and the endogenous growth model is similar to Pagano's (1993) model . The two sectors in the loan model operate side by side, while borrowers and interest rates are different.

Banks do not impose any restrictions on the availability of loans (there is no a binding supply constraint). The only limitation is that borrowers need to use the loans on pre-specified projects. As a result of a maximization process, borrowers decide which project they will undertake.

Our financial sector initially has two **separated financial markets**. The distinction of lenders is not important, and the same lender could provide resources to both markets. What is important here is the distinction between borrowers. There are two types of borrowers (firms). We assume, that both types of firms do not use self-financing.[2] All agents are profit maximizers. Banks compete among themselves. Credit suppliers are concerned about the interest rate they charge and the riskiness of borrowers.

Both lenders and borrowers have identified a number of projects, with different levels of return. Projects are indexed by their gross return. $R_1 < R_2$. $R_i = (1 + \phi_i)$. Where ϕ_i is the rate of return of project i. R_1 represents a group of projects with low expected returns and low risk. For our analysis we will assume that they are risk-free. On the other hand, projects type 2 are risky. The loans of type 2 are labeled L_2. The total of loans in the economy is:

$$L_T = L_1 + L_2 \qquad (1)$$

Banks can distinguish if a borrower invests in project type 1 or type 2. The payoff of project 1 is R_1 with certainty. On the other hand, if borrowers invest in type 2 projects, their payoff is a random variable, i.e. project 2 pays R_2 with probability δ and 0 with probability $1-\delta$. To make our life easier we will assume that this probability is constant[3]. Thus δ is the probability of loan repayment. The loans are fixed-rate for the two types of projects, type 1 projects have an interest rate of r_1 , and type 2 of r_2.

The availability of funds depends on the amount of deposits that lenders receive from the public, D and their own equity, E. We will assume E=0 without loss of generality. Thus,

$$L_T = (1-\pi)\,D + E \qquad (2)$$

where $\pi < 1$,

and $(1-\pi)$ is the ratio loans-to-deposits, given that E=0. The constant π includes the legal reserve requirements, the reserves the banks keep to cover for the depositor's withdrawals, and their costs to transform the deposits into loans. In other words, the more efficient the banks the smaller π. The ratio loans-to-deposits depends on the probability of repayment because banks, i.e. $\pi = f(\delta)$, as δ increases π will decrease. But in this section of the paper we will assume δ as fixed.

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[1] The number of sectors in the financial market is not relevant.

[2] We assume this because we just care about the financing provided by banks.

[3] In a more realistic model the lender by the process of credit analysis can assign a probability of default to each borrower.

Lenders are risk-averse, because they expect higher returns when they provide financing for type 2 loans. Santomero (1984), Hannan (1991) and Startz (1983) used similar functions. Now we have all the elements to determine the price of Loans. Lenders maximize their expected profits[4], that is:

$$\underset{\{L_1, L_2\}}{\text{Max.}}\ \Pi^e = L_1 r_1 + \delta L_2 r_2 - D r_d \quad (3)$$

$$\text{s.t. } D = \omega(L_1 + L_2), \quad (4)$$

$$\omega > 1;\ \omega = 1/(1-\pi) \quad (5)$$

where: r_d is the exogenous interest rate paid to depositors. Each bank is considered to be small in the market and unlimited liability of bank shareholders is assumed.

Substituting the constraint into the objective function, the first order conditions (FOC) are the following:

$$\partial\Pi^e/\partial L_1 = [\partial(L_1 r_1)/\partial r_1]*[\partial r_1/\partial L_1] - r_d\,\omega = 0 \quad (6)$$

$$\partial\Pi^e/\partial L_2 = [\partial(\delta L_2 r_2)/\partial r_2]*[\partial r_2/\partial L_2] - r_d\,\omega = 0 \quad (7)$$

From the FOC we can get the interest rate charged for every type of loan:

$$r_1 = (r_d\,\omega)*\varepsilon_1 \quad (8)$$

$$r_2 = (r_d\,\omega)*\varepsilon_2/\delta \quad (9)$$

Where: ε_i is the elasticity of loan i supplied with respect to changes in the interest rate i. Thus the interest rate charged for each type of loan depends on the bank's costs, the ratio loans-to-deposits, the return paid by deposits, and the repayment probability. When this probability is larger the bank will charge less for type 2 loans.

We can get the relationship between the interest rates charged for different categories of loans, dividing (9) by (8) and assuming that elasticities in both markets are the same:

$$r_1 = \delta * r_2 \quad (10)$$

It is easy to see that an increase in the exogenous δ (probability of loan repayment)-ceteris paribus- will make the difference between r_1 & r_2 become smaller, that is the spread between these two rates will narrow.

Assuming that all lenders are homogeneous, i.e. every bank maximizes its profits in the same way, in equilibrium, they all end up providing the same amount of credit. Also, the interest rates, the default probability, and the loans-to-deposits ratio are all exogenous to banks. Thus the total supply of loans for each type of loans is, $L_i = n*L_i$, where n is the number of lenders. Consequently, with the usual downward sloping demand curve there will be an equilibrium interest rate for every type of loan, and supply and demand will be equal in each market.

Turning to the demand side of the market, we will analyze the maximization process of borrowers. All borrowers are price takers, because they are small and cannot influence prices. There are two types of borrowers, type 1 and type 2, both are risk-averse, so they dislike risk. Type 1 borrowers are more risk-averse than type 2. Both are utility maximizers. Let us assume that both types of borrowers only care about the expected return and risk of returns (Variance).

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[4] Expected profits are assumed to be a concave function.

The profits for borrowers, P, are a function of the return of the project R_i, the interest rate they pay to lenders, r_i, and other costs of the project, C. We will assume that all other cost are zero, i.e. C=0, and that projects are fully loan-financed.

$$P_i = R_i * Li - (1+r_i) * Li \qquad (11)$$

Where: Li = amount of the loan, and also the amount that should be invested in the project.

Dividing both sides of the previous equation by Li, we get the profit rate for project i, p_i. Thus:

$$p_i = [R_i - (1+r_i)] = \phi_i - r_i \qquad (12)$$

As projects returns are stochastic, the profits are actually expected profits. For Project 1 the payment occurs with certainty, but for Project 2, the payments are R_2 with probability δ and zero with probability $(1-\delta)$. Borrowers maximize their expected utility, which is assumed to be of the form:

$$E(U_s)^i = E(p_i) - k_s * V_i, \qquad (13)$$

where: k_s = constant that indicates the degree of risk aversion;

V_i = variance of profits (constant).

Note that subscripts i indicates projects type i, and s indicates the borrower's type.[5]

<u>For borrowers type 1</u>, if they invest in projects type 1, their profits are certain, i.e. they are not stochastic. The variance is zero, $V_1 = 0$. So,

$$E(p_1) = p_1 = \phi_1 - r_1; \qquad (14)$$

Consequently the expected utility for project type 1 is:

$$E(U_1)^1 = \phi_1 - r_1 - k_1 * V_1 = \phi_1 - r_1 \qquad (15)$$

When they invest in projects type 2, their profits are stochastic, thus returns are in terms of expected values, and their variance[6] is greater than zero; that is,

$$E(p_2) = \delta(\phi_2 - r_2) \qquad (16)$$

As $V_2 > 0$, the expected utility for type 2 projects is,

$$E(U_1)^2 = \delta(\phi_2 - r_2) - k_1 * V_2, \quad \text{or}$$

$$E(U_1)^2 = \delta(\phi_2 - r_2) - k_1 * [(\phi_2 - r_2)^2 \delta(1-\delta)] \qquad (17)$$

<u>For type 2 borrowers</u>, the corresponding equations are,

When they undertake type 1 projects:

$$E(p_1) = p_1 = \phi_1 - r_1; \qquad (18)$$

$$E(U_2)^1 = \phi_1 - r_1 - k_2 * V_1 = \phi_1 - r_1 \qquad (19)$$

When they undertake type 2 projects:

$$E(p_2) = \delta(\phi_2 - r_2) \qquad (20)$$

$$E(U_2)^2 = \delta(\phi_2 - r_2) - k_2 * [(\phi_2 - r_2)^2 \delta(1-\delta)] \qquad (21)$$

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[5] Here k_1 represents all values of the degree of risk aversion that make borrowers type 1 to choose project 1, and k_2 represents the sufficiently small values that allow agents type 2 to choose project 2.

[6] The Variance for Project 2 is $V_2 = E[R_2 - E(R_2)]^2 = (\phi_2 - r_2)^2 (1-\delta)\delta$, as the probability of repayment increases the variance becomes smaller.

As both types of borrowers are utility maximizers, they will choose the project that gives them the higher expected utility.

For borrowers of type 1, when:

$$k_1 > [\delta(\phi_2 - r_2) - (\phi_1 - r_1)] / [(\phi_2 - r_2)^2 \delta(1-\delta)] \quad (22)$$

$E(U_1)^1 > E(U_1)^2$ i.e., the expected return of project 1 is greater than the one for project 2. Their degree of risk aversion is high and the negative influence of the variance in their utility function is high. Hence it over-compensates the differences in expected utilities between projects.

On the other hand, with k_2 sufficiently small borrowers of type 2 prefer type 2 projects, i.e., $E(U_2)^1 < E(U_2)^2$. This is because the influence of the variance is negligible, so they are highly influenced by the expected returns. The correspondent values for k_2 are:

$$k_2 < [\delta(\phi_2 - r_2) - (\phi_1 - r_1)] / [(\phi_2 - r_2)^2 \delta(1-\delta)] \quad (23)$$

As a result of the process of utility maximization, borrowers that are more risk-averse will select the "safe" projects. On the other hand, the less risk-averse will opt for the risky projects. There will be a market for the safe and the risky projects, with different interest rates. In other words, the distribution of loans between the different classes of projects is based on the differences in borrowers' risk aversion, as well as the riskiness of projects.

Note that the increase in the probability of repayment or decrease in the risk has a twofold effect. First, it decreases the spread in the interest rate between the risky and the safe projects; and second it makes borrowers less risk-averse. In other words, they are more likely to undertake projects with higher expected return.

In the literature of credit rationing the probability of default augments with the size of loans [Stiglitz and Weiss (1981)]. Therefore with the increase in the loan's size lenders need to charge a higher interest rate to compensate for the additional risk. Although after some point, the default probability becomes so high that no matter how high the interest rate charged is, that the supply of loans does not increase anymore. Eventually the slope of the of the supply becomes negative as the marginal cost is higher than the marginal revenue. In other words, no matter how high the interest rate is, lenders will not increase the loans supply. This model is consistent with that situation, but the negatively sloped supply does not arise because we assume a constant default probability[7], and the size of each loan is pre-determined.

Default probability and the role of Government[8]

Before explicitly incorporating a model for the default probability, we will provide some comments about the risks faced by depository institutions. These risks are refinancing risk, reinvestment risk, and credit risk. Refinancing risk designates the fact that the cost of rolling over or re-borrowing funds will rise above the returns being earned on asset-investments. Reinvestment risk applies to the event when returns on funds to be reinvested fall below the cost of funds. And finally, the default risk arises because borrowers may or may not pay in full their loans. Again we can distinguish between the specific default risk (micro risk) and general default risk (systematic risk). Specific risk refers to the one identified with a particular type of project and differs from firm to firm. On the other hand, the risk of default associated with the

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[7] Although with increasing default probability with the size of loans increasing we can get a segment of the supply curve that is downward sloping.

[8] This section is based on a previous paper prepared for the Econ450 class "Financial Markets in a Developing Economy: Role of the Public Sector".

whole economy or the macroeconomic conditions affecting all economic agents, is called systematic risk.

We will concentrate in this last type of credit risk, the systematic one. It is here where governments can have a positive and unique role on reducing it. The nature of actions that are needed to reduce this type of risk are those of a public good.[9] A characteristic of public goods is that under competitive circumstances they are provided in a lower quantity than socially optimal. This is because of the "free rider" problem, which refers to the fact many individuals will fail to contribute to the cost of the good they are using because they will get the good's benefit once it is provided by others. This situation arises due to the fact that some kinds of investments generate benefits not only to the investor, but to the whole community given the "non-exclusive" characteristic of the good or service created.[10]

Accordingly we will relax the assumption of constant repayment probability, and prepare an explicit sub-model for it. In this course of actions we could use qualitative or quantitative information about borrowers, to assess the probability of repayment and assuming some kind of distribution for this probability. In this section, we will follow a more general approach based on the learning by doing and knowledge spillovers literature. The general assumption is that investments in the financial sector, can decrease the general risk for loans. This is especially relevant for infant financial markets in many LDC's. We apply this approach following Barro and Sala-I-Martin (1995).

We will incorporate the learning by doing is throughout each financial institution investment and government spending. Government spending especially refers to modernization of the regulatory and institutional framework. For example, mandatory disclosure of reliable information about firms and financial intermediaries may enhance confidence of depositors in banks and facilitate the monitoring of firms by banks. Improvements in accounting procedures according to international standards may provide more confidence in banks, improve efficiency, and facilitate foreign investment.

Also, enhancement in efficiency will lead to an increase in the stock of knowledge in the sector. In addition, we assume that there exist spillover of knowledge, i.e., if one firm uses an idea, then it does not prevent other companies from using it.[11]

The default probability or credit risk can be written as the inverse of the repayment probability in the following way:

$$\Lambda = 1 / \delta \qquad (24)$$

$$\Lambda = \Lambda_1 + \Lambda_2 \qquad (25)$$

Where: Λ = total credit risk, as perceived by banks

Λ_1 = systematic risk

Λ_2 = micro risk

We will not do anything about the micro risk because it can be eliminated by diversification. The systematic risk behavior can be captured using this equation:

$$\Lambda_1 = q\, A^{-w}\, g^{-h} \qquad (26)$$

$$A = k_p + g$$

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[9] By public good we understand one that provides benefits to all individuals at the same time and whose utilization by one person is no way diminished by that of another (Todaro, 1994).

[10] In other words, a free rider is anyone whose contribution to the cost of production of a good or service is less than the marginal value he derives from it. In addition, in the literature free riding also refers to the failure of individuals to reveal their true preference for the public good by their contribution to produce it.

[11] We think that this is a good assumption because in LDC's the patents and inventions are not enforced as in the industrialized countries.

where: A = indicator of knowledge or capital in the financial sector.
g = government expenditure in modernization of the financial sector
q = initial level of systematic risk (positive constant).
k_p = private sector investment in the financial sector.
w, j, h positive constants.

Therefore, when government invests in the financial sector reduces Λ_t. In addition, this investment increases A, that provides a spillover benefit to all banks in the financial sector. Also when firms invest in modernization of their systems of information or their human capital they can get better assessment of their loans risk, reduce their costs, and increases A, which in turn reduces Λ_t further. We do not include as an additional factor for simplicity and because the effect will be the same as in the case of the government variable.

The government's role here could be very important, i.e., it could modernize the legal framework for the financial institutions and eliminate barriers to different operations. In addition, it could impose better fiscalization for all businesses, improve the accounting systems, promote a capital market (stock market, derivatives market, etc.). In general, these and any similar kinds of investment could promote a well functioning financial system.

Economic Growth, Government and Probability of Loans Repayment

The dynamics of government investment is the following: initial investment in modernization of the financial sector,[12] which according to equation (26) will decrease the default probability of the overall economy. Then this decrease in the overall probability of default implies a decreased Λ (equation 25). Throughout equation (24) the repayment probability will be increased. By (10) banks can decrease the interest rate charged for the risky projects. Assuming a downward sloping investments demand, the lower interest rate will imply higher investments. Also investors will become less risk averse (equations 22 and 23), thus they would be willing to undertake the higher return projects. The latter will improve the economy's resource allocation and move up the long-run growth rate. Furthermore, additional external resources can become available (foreign investment and external loans). "Foreign investment can contribute substantially in order to complement the internal capital formation, acting as a provider of technology and 'know how'...."(Insfrán,1995), which could accelerate growth. In other words, financial resources will be increased. These increases in investment will indicate an rise in the level and growth rate of real GDP, ceteris paribus.

To show that a decrease in the probability of default will have an impact in the long-run growth rate of the economy, we will use a very simple endogenous growth model, following Pagano (1993). The model is the "AK" model, output depends linearly from the aggregate capital stock. Here the capital is taken in a broad sense, as a composite that includes physical and human capital, similar to Lucas (1988).

$$Y_t = A\ K_t \qquad (27)$$

where: A is the social marginal productivity of capital.

To simplify let us assume that population is constant and that the economy produces a single good that can be invested or consumed. There are two technologies to produce the single good and they are the ones that we call at the beginning of the paper as risky and safe projects.
There exists a depreciation rate Ω, associated with the investment. Then Gross investment equals

[12] Suppose the government get the resources by contracting a foreign loan.

$$I_t = K_{t+1} - (1- \Omega) K_t \qquad (28)$$

In a closed economy, in equilibrium we require that Gross Investment equals Gross Savings. We assume that all savings are deposited into banks, and Investments are equal to total loans, i.e.,

$$S = D \qquad (29)$$

$$I = L_T \qquad (30)$$

From equation (2), with E=0, we have $L_T = (1 - \pi) D$, and using (29) and (30) we get the following relationship,

$$I = (1- \pi) S \qquad (31)$$

Equation (31) implies that a proportion π of Savings is "lost" in the intermediation process.

The growth rate of this economy is:

$$\psi_{t+1} = (Y_{t+1} - Y_t) / Y_t = (K_{t+1} - K_t) / K_t \qquad (32)$$

The steady-state growth for this model is:

$$\psi = A\, I / Y - \Omega = A (1-\pi) s - \Omega \qquad (33)$$

where s= S/Y, the savings rate.

We can easily see in (33) that a decrease in the default probability will influence the long-run growth rate of the economy, by increasing Investments, I , and the ratio loans-to-deposits, (1-π), which will also be increased. Therefore the financial sector is a factor that determines the growth rate in the economy.

In sum, in this section we have shown that the financial sector (here the loans sector) can have a role in the long-run growth of the economy. More precisely, the influence will come by changes in the probability of loans repayment. In turn, governments can help increasing this probability by modernizing the institutional and regulatory framework in the financial markets. In addition, an improvement in the efficiency of banks will be reflected in an increase in (1-π) will also generate higher rates of economic growth in the long run.

CONCLUSION

This model explains interest rate differentials in the loans market based on the riskiness of projects and the risk aversion of borrowers. These differences emerge simply from the optimization of lenders and borrowers. Therefore, given the ratio loans-to-deposits, interest paid to depositors and the probability of repayment, we provide an optimal rule for banks to price their loans. The relationship between the two interest rates depends on the riskiness of projects. Therefore, one way to reduce interest rates could be to reduce the overall risk of projects in the economy (increase probability of repayment).

The default probability is related the risk faced by banks and to the efficiency of the financial system in transforming deposits into loans, or savings into investments. We have argued that government investments in the financial markets could improve the financial markets' performance and facilitate the transfer of funds from savers to investors. In other words the systematic risk can be reduced by the institutional changes promoted by the government. This is especially relevant because of the characteristic of public good of government investment in the financial sector. Also, a good regulatory framework can promote efficiency, solvency, and security to financial intermediaries.

All these benefits ultimately will increase the long-run growth rate in the economy through the improvement in the allocation of financial resources, an expansion in investments, and, an increase of the ratio loans-to-deposits. Therefore as a way to promote growth, less developed countries should promote the development of their financial systems.

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