A BVAR MODEL FOR FORECASTING PARAGUAY'S INFLATION RATE IN TURBULENT MACROECONOMIC ENVIROMENTS

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Abstract

In this research I explore the methodology of Bayesian autoregressive methods to forecast inflation and other macroeconomic time series of interest. I estimate a Bayesian vector of autoregressive model to forecast inflation, GDP and the interest rate of Paraguay taking as main approach the Minnesota prior methodology developed by R.B. Litterman (1984). The main out of sample accuracy statistics, the RMSFE and U-Theil statistic results show that in the 75% of the subsamples of forecast characterized as turbulent macro environments, Bayesian specifications outperform traditional VAR models in terms of accuracy. When using quarterly data Bayesian techniques deliver also more accurate forecasts than VAR models ones.

1. Introduction

In this research I study Bayesian inference methods in order to contribute to Paraguayan Central Bank monetary policy forecasts and to the growing literature of Bayesian forecasting. The basic idea here is that given the scarcity of data to forecast macroeconomic time series with common unrestricted vector of autoregressive (UVARs), priors about the probabilistic density of the parameters of interest can be used to outperform the original UVAR models in terms of accuracy as in Koop and Korobilis (2010). In particular what I do is to compare the accuracy of the forecast results of Minnesota Prior elicitation approach in a BVAR modelling environment versus the results obtained with a VAR using the RMSFE as measure to determine whether should be preferable to forecast with Bayesian vector of autoregressive models (BVAR). The main finding of this paper is that Bayesian techniques improve the performance of traditional VAR models in the 75% of turbulent subsamples used to check forecast accuracy.

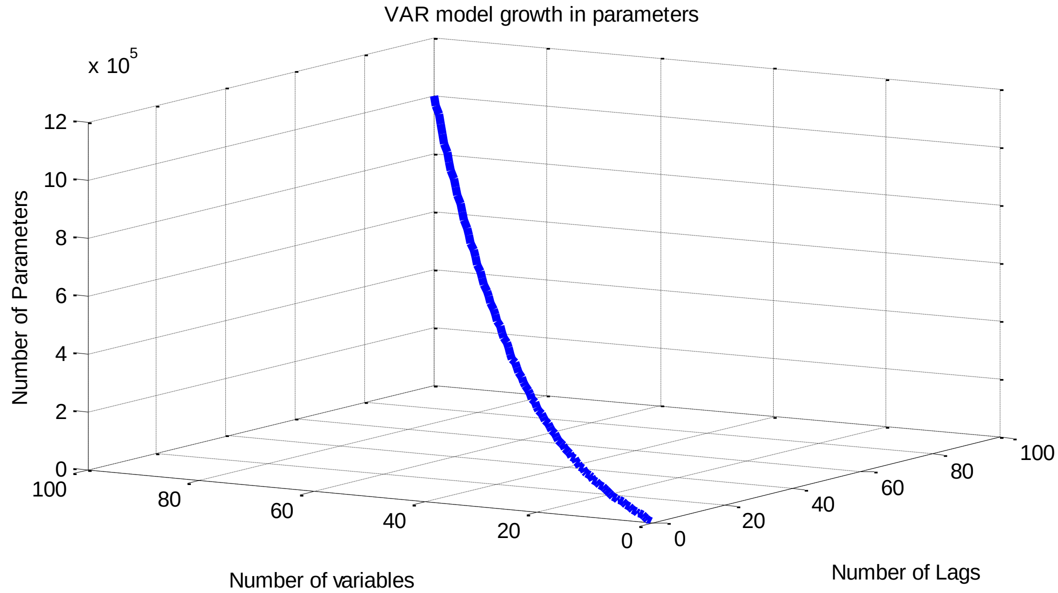
2. BVAR modelling approach

Bayesian Vector of Autoregressive modelling approach provides a general method for combining a modeller's beliefs with the evidence contained in the data (i.e see Hamilton (1994)). In contrast to the classical approach based on estimating a set of parameters, Bayesian statistics presupposes a set of prior probabilities about the underlying parameters to be estimated.

A recent compilation of related literature has mentioned (see Koop and Korobilis (2010)), traditional VAR and Bayesian techniques to forecast macroeconomic time series differ in relation to three issues:

First, VARs are not parsimonious models. They have a great number of coefficients. For instance, α_{Mn} contains KM parameters, which for simple a VAR(4) with five variables is 105. An illustrative graph is attached below showing the pattern of increase in the number of parameters of a VAR model is exponential.

Graph 1: VAR model patameters



Source: Own elaboration

With quarterly or monthly macroeconomic data to get robust estimates of the parameters the amount of data needed for each variable might be at most a few hundred. Without prior information it is hard to obtain precise estimates of so many coefficients and thus, forecasts such as impulse responses will tend to be imprecisely determined. For this reason it can be desirable to shrink forecasts and Bayesian techniques adding prior information offers a sensible way of doing shrinkage.

Second, the priors used with BVARs differ in whether they lead to analytical results for the posterior and predictive densities or whether Monte Carlo Markov Chain (MCMC) methods are required to carry out Bayesian inference. In BVARs, natural conjugate priors lead to analytical results, which can greatly reduce the computational burden.

Third, the priors differ in how easily they can handle departures from the unrestricted VAR such as allowing for different equations to have different explanatory variables, allowing the coefficients to change over time, etc.

Working with Bayesian models implies updating our beliefs after seeing the data. For example, one might have a strong prior that the first autoregressive coefficient in an AR(p) model for the inflation is equal to unity and that all other coefficients are zero. Such a prior would be consistent with the view that the inflation rate follows a random walk or that changes in the inflation rate are completely unpredictable.

Bayesian estimation of the parameters of a VAR(p) model will revise this prior view in the light of the empirical evidence contained in a time series model of inflation and other variables. A prior hypothesis about a particular parameter value can be confirmed by any observation which is likely given the truth of the prior hypothesis. This contrasts significantly with classical approaches to parameter estimation such as the maximum likelihood where one chooses as point estimates values such that the likelihood of obtaining the actual sample of data being maximized regardless of any prior probabilities which are or could be assigned to the parameters.

Bayesian econometrics makes use of equation (1) to perform statistical inference analysis:

(1) $P(\theta \mid y) = \dfrac{P(y \mid \theta) P(\theta)}{P(y)}$

Where the data is represented by y and the BVAR model parameters are represented by θ.

For Bayesian econometricians the object $P(\theta \mid y)$ is of fundamental interest. That is, it directly points out the question: given the data, what do we know about θ? The treatment of θ as a random variable is controversial and it is based on a subjective view of what probability is. Established that we are interested in $P(\theta \mid y) = \dfrac{P(y \mid \theta) P(\theta)}{P(y)}$ we can rewrite (4) as:

(2) $P(\theta \mid y) \propto P(y \mid \theta) P(\theta)$,

where $P(\theta \mid y)$ is the posterior, the fundamental density of interest that tells us everything we know about θ after seeing the data, $P(y \mid \theta)$ is the likelihood function and one could think about it as the data generating process and $P(\theta)$ is the prior representing all information we have about θ and that is not contained in the data.

In addition to learning about the parameters of a model an econometrician might be interested in comparing different models. A BVAR model is formally defined by a likelihood function and a prior. Hence the posterior of the parameters calculated using M_i is written as:

(3) $$P(\theta^i \mid y, M_i) = \frac{P(y \mid \theta^i, M_i) P(\theta^i \mid M_i)}{P(y \mid M_i)}$$

The logic of Bayesian econometrics suggests that we use Baye's rule to derive a probability statement about what we do not know (i.e, whether the model is correct or not) conditional on what we do know (i.e, the data).

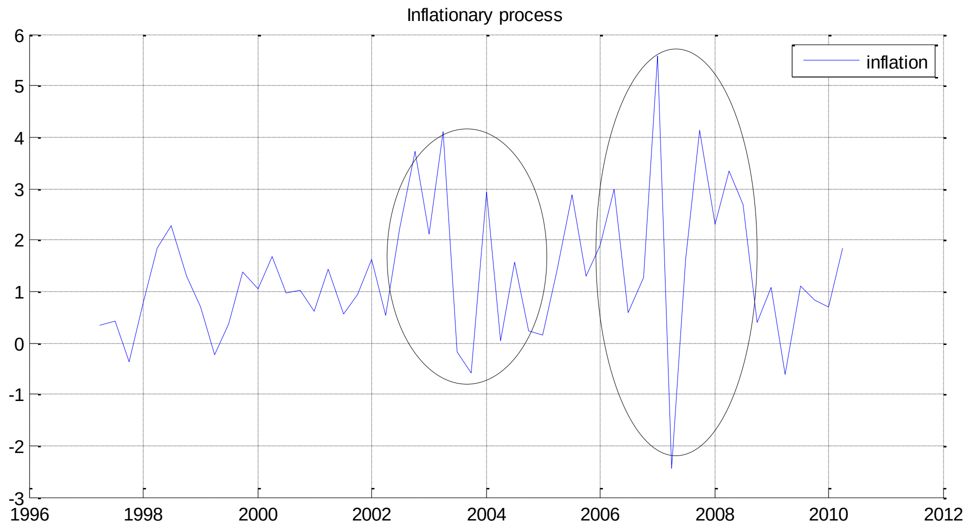
3. Methodology

The graphical visualization of the data shows two main highly turbulent time intervals located in the periods of 2002M01-2003M01 and 2003-2004M01 and 2006:04:2007:04 and 2007:05-2008:05 where inflation rises up to a 5% in a month and reaches the historical bottom of a -2.2% variation in a month. Whether using linear methods to forecast in turbulent times is recommendable or not is not an issue in this research. However, it is an issue to determine whether should be preferable to forecast monthly inflation with Bayesian vector of autoregressive models rather than with traditional VARs.

Hence I first estimate a VAR model containing in the vector y_t variables such log(GDPt), πt and Interest rate (rt) using the Theil-Goldeberg mixed regression procedure for monthly frequency series. Second, I use the estimated posterior densities for the BVAR model and the traditional UVAR estimated coefficients to implement direct forecasts h periods ahead and compute the

RMSFE to determine which model was more useful for monetary policy decisions in the moments turbulences remarked before that can be seen in graph 2.

Graph 2: Monthly inflation rate in %



Source: Own elaboration with BCP data

The VAR (p) model can be written as:

(4) $y_t = a_0 + \sum_{j=1}^{P} A_j Y_{t-j} + \varepsilon_t$

Where yt for t=1....,T is an Mx1 vector containing observations on M time series variables, ε_t is an Mx1 vector of errors, a_0 is an Mx1 vector of intercepts and A_j is an MxM matrix of coefficients. I assume ε_t to be i.i.d N[0, Σ]. The VAR model can be written in different ways and depending on how this is done, some of the literature expresses results in terms of Multivariate Normal and others in terms of the matric-vriante Normal distribution (see Canova (2007) or Koop (2010)). The latter arises if Y is defined to be a TxM matrix which stacks the T observations on each dependent variable in columns next to one another. ε and E denote stackings of the errors in a manner conformable to y and Y respectively. Define

$x_t = (1, y'_{t-1}, ..., y'_{t-p})$ and $X = \begin{bmatrix} x_1 \\ x_2 \\ .... \\ x_T \end{bmatrix}$. Note that, if we let K=1+Mp be the number of coefficients in each equation of the VAR then X is a TxK matrix. Finally if A=(a_0 , A_1,...A_p)' we define $\alpha = vec(A)$ which is a KMx1 vector which stacks all the VAR coefficients into a vector. With all these definitions, we can write the VAR either as:

(5) Y=XA+E or

(6) $y_t = (I_M \otimes X)\alpha + \varepsilon$ where $\varepsilon \sim N[0, \Sigma \otimes (X'X)^{-1}]$.

The usual procedure to determine the lag length (P) of the independent variables is to use penalty methods. Penalty function statistics such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Hannan-Quinn(HQC) among others have been used to assist time series analysts in reconciling the need to minimize errors with the conflicting desire for model parsimony. These statistics all the take the form of minimizing the sum of the residuals sum of squares plus a 'penalty' term which incorporates the number of estimated parameters coefficients to factor in model parsimony. For the monthly dataset I run the tests obtaining that for the first subsample of , 2002-2003, lag length criteria such as AIC, BIC and HQC suggest the use of models as a BVAR(1) and BVAR(20). For the second sample that ranges from 2003-2004 I found the models that minimize the loss of information are a BVAR(23) and a BVAR(1). For the forecast period 3 that ranges from 2006:05-2007:05 I find that the best model to forecast is a BVAR(14). The last out of sample subsample for forecast in which I carry out analysis ranges from 2007:05 to 2008:05 and the model used to forecast is a BVAR(12) as recommended by all the information criteria: AIC, BIC and HQC. All statistics of these entropy minimization tests can be found in the Appendix in tables JJ to P

When working with quarterly data, since the purpose is to elucidate the superiority of the Bayesian method I will estimate 8 versions the system connecting the variables log(GDPt), log(interest rate) and πt for for each lag length ranging from p=1 to 8 and I will just use one subsample of forecast to measure the accuracy since the quarterly data set is quite small.

The estimation of the BVAR reported in this paper is based on Theil's (1963) mixed estimation technique. Mixed estimation is a relatively simple and intuitive means of combining sample information witch stochastic prior information. Suppose that we have m priors which we wish to take account in deriving Bayesian estimates of the parameters of the VAR model. The idea is to estimate a regression with N+m observations: N of them corresponding to the information in the sample and m of them corresponding to the restrictions. The m observations corresponding to the restrictions are weighted relative the observations in the sample according the degree of tightness in each prior. As the amount of information in the prior tends toward zero, for an extremely diffuse prior, the mixed estimators of the parameters of BVAR tend toward the OLS estimates of the parameters of UVAR

Once posterior estimation is performed I test the forecast accuracy of the models. I carry out recursive forecasting exercise using the direct method. That is, for $\tau = \tau_0, ..., T - h$ I obtain the predictive density of $y_{\tau+h}$ using data available through time τ for h=1 and 4. τ_0 is 1994M12. I will use the notation where $y_{i,t+\tau}$ is a random variable we are wishing to forecast (CPI, IMAE and Interest rate), $y^0_{i,t+\tau}$ is the observed value of $y_{i,t+\tau}$ and $p(y_{i,t+\tau} \mid data_\tau)$ is the predictive density based on information available at time τ. I use the direct forecast method although it has been shown by J.H.Stock, Massimiliano Marcellino and Mark.W.Watson (2005) that iterated forecasts tipically outperform direct methods. The point of my choice is that in theory (see direct forecasts are more robust to model misspecification that iterative ones. Concretely, the RMSFE formula is:

$$(8)\quad RMSFE(n) = \sqrt{(T)^{-1} \sum_{i}^{T} \left(y_i^a - y_i^f(n) \right)^2}$$

The U-Theil Statistic is the other common measure to determine models accuracy:

(9) $$U - Theil = \frac{\sqrt{(T)^{-1}\sum_{i}^{T}\left(y_i^a - y_i^f(n)\right)^2}}{\sqrt{(T)^{-1}\sum_{i}^{T}\left(y_i^a - y_i^*(n)\right)^2}}$$

4. Minnesota prior implementation

Early work in the shrinkage of priors was done by researchers at the University of Minnesota and the Federal Reserve Bank of Minneapolis. The prior Litterman R.B (1984) used has come to be known as Minnesota prior. It is based on an approximation which leads to great simplifications in prior elicitation, to substitute Σ by $\hat{\Sigma}$. The original Minnesota prior simplifies further by assuming Σ to be a diagonal matrix. When Σ is replaced by $\hat{\Sigma}$ we only have to worry about a prior for α and the Minnesota prior assumes:

(10) $\alpha = vec(A) \sim N[\underline{\alpha}_{Mn}, \underline{V}_{Mn}]$.

To explain the Minnesota prior note first that the explanatory variables in the VAR in any equation can be divided into the own lags of the dependent variable, the lags of the other dependent variables and exogenous or deterministic variables. For the prior mean $\underline{\alpha}_{Mn}$, the Minnesota prior involves setting most or all of its elements to zero and ensuring shrinkage of the VAR coefficients toward zero and lessening the risk of over-fitting. Prior statements can be expressed mathematically using the set of univariate prior density functions $g(\alpha_1), g(\alpha_2), ..., g(\alpha_P)$ of the autoregressive parameters $\alpha = \{\alpha_1, \alpha_2, ..., \alpha_P\}$ The random walk prior which one might wish to take account when estimating this model is that the mean of α_1 is equal to unity whereas the means of all other autoregressive parameters of higher order than one are zero.

$$E[\alpha_1] = \int_{-\infty}^{\infty} \alpha_1 g(\alpha_1) d\alpha_1 = 1$$

(11) $$E[\alpha_2] = \int_{-\infty}^{\infty} \alpha_2 g(\alpha_2) d\alpha_2 = 0$$

....

$$E[\alpha_P] = \int_{-\infty}^{\infty} \alpha_P g(\alpha_P) d\alpha_P = 0$$

The original R.B Litterman or Minnesota prior (1984) was based on the idea that each series is best described as a random walk around an unknown deterministic component. Hence the prior distribution is centered around the random walk specification for variable n given by:

(11) $y_{n,t} = \mu_{n,t} + y_{n,t-1} + \varepsilon_t$

According to this specification, the mean of the prior distributions on the first lag of variable *n* in the equation for variable n is equal to unity. The mean of the prior distribution on all other coefficients is equal to zero. Of course, if the data suggest that there are strong effects from lags other than the first own lag or from the lags of all the other variables in the model this will be reflected in the parameter estimates.

Once the means haven been specified, the only other prior input is some estimate of the dispersion about the prior mean. As described by Litterman (1984), the standard error on the coefficient estimate for lag *l* of variable *j* in equation *i* is given by a standard deviation function of the form:

(12) $S(i, j, l) = \frac{[\gamma g(l) f(i, j)]}{s_j} s_i$ where

(13) f(i,j)=1 if i=j and w_{ij} otherwise

The 'hyper-parameter'' γ and functions $g(l)$ and $f(i, j)$ determine the tightness or weight attaching to the prior above. The term γ is also called ''overall tightness'' of the prior. Function g(l) determines the tightness on lag one relative to lag l. The tightness around the prior mean is normally assumed to increase with increasing lag length. This is achieved by allowing g(l) decay harmonically with decay factor d, i.e, $g(l) = l^{-d}$. The tightness of the prior on variable *j* relative to variable *i* in the equation for variable i is determined by the function f(i,j). This can be the same across all equations in which case w_{ij} is equal to a constant w and the prior is said to be symmetric. Finally the multiplicative ratio s_i/s_j in equation (12) reflects the fact that in general the prior can not be completely specified without reference to the data. In particular it corrects for differences in the scale used in the measurement of each variables included in the system. The Minnesota prior assumes the prior covariance matrix $\underline{V}_{Mn}$ to be diagonal and the posterior has the form of:

(14) $\alpha \mid y \sim N[\bar{\alpha}_{Mn}, \bar{V}_{Mn}]$ where

(15) $\bar{\alpha}_{Mn} = \bar{V}_{Mn}[\underline{V}_{Mn}^{-1} + (\hat{\Sigma}'^{-1} \otimes X)' y]$ and

(16) $\underline{V}_{Mn} = [\underline{V}_{Mn}^{-1} + (\hat{\Sigma}'^{-1} \otimes (X' X))]^{-1}$

5. Results discussion

I estimate the model with the Theil-Goldberg (1971) procedure taking all the data I can into consideration and I elicitate the Minnesota prior as commented before for the different subsamples of interest which characterized for a high uncertainty associated to them. The concrete parameterization of the Minnesota prior I use is the following one:

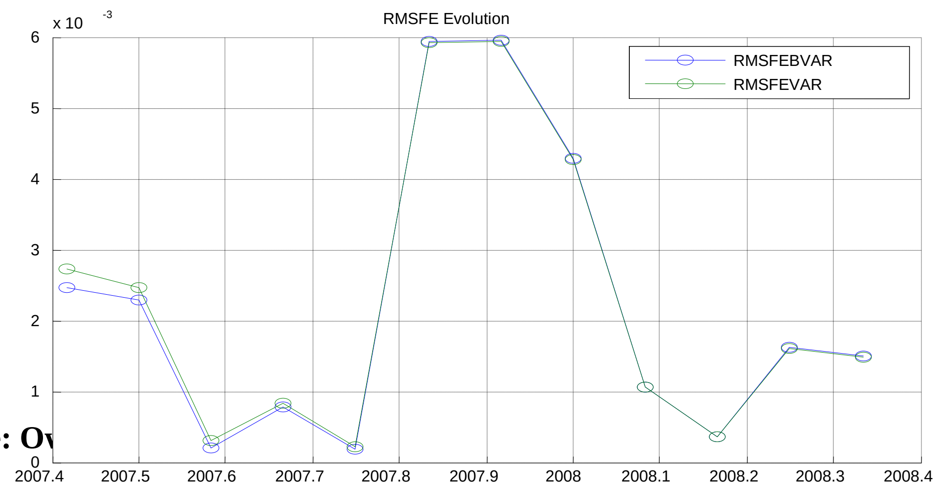
Tightness =0.8;
Decay = 0.35;
Weight = 0.01;

The results obtained from the house race between the VARs specifications and the different Bayesian models for the periods of 2002M01-2003M01 and 2003-2004M01 and 2006:04:2007:04 and 2007:05-2008:0 show the improvement effect in accuracy Bayesian models due to shrinkage techniques of the coefficients what allows BVAR outperform with both monthly and quarterly data the results of VARs when forecasting.

Forecast subsample 1

The implementation of the Bayesian approach in this subsample with the former prior yields with p=1, a slightly lower RMSFE for the BVAR at the first 2 horizons of the forecast and replicating the results of the UVAR for the rest of the period as it can ben seen in graph 3.

Graph 3: RMSFE BVAR(1) VS VAR(1)



Source: Ov

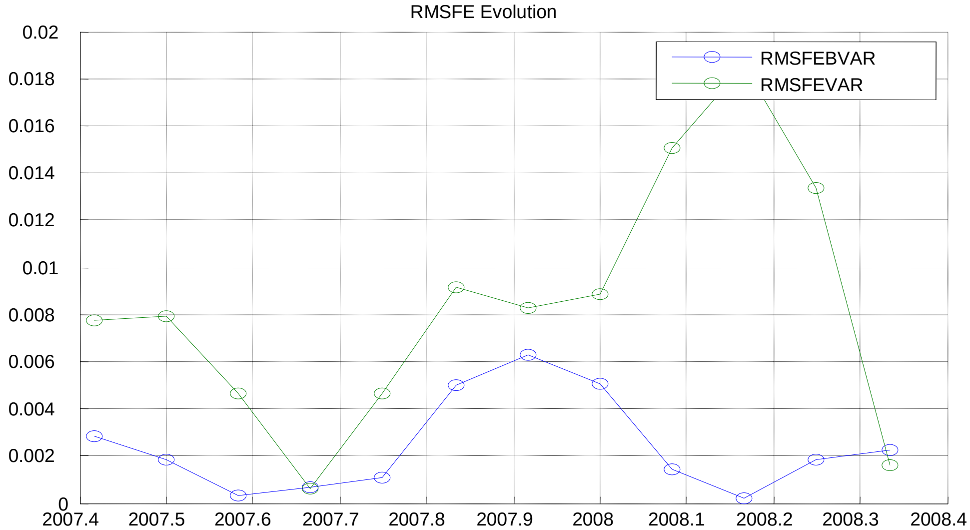
The results over all the variables captured in the U-Theil's matrix show that the model performed relatively well when forecasting inflation rate, improving imae's and interest rates forecast results for the case of the BVAR(1).

Table 1: U-Theil BVAR(1)/VAR(1)

BVAR(1)	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11	h=12
U-Theil Imae	0,9743	0,92524	0,64706	0,889	0,804	3,1774	0,9092	1,1702	1,1033	1,09	1,1	0,9
U-Theil Int rate	0,6546	0,45333	1,02114	4,563	1,415	2,1963	1,0921	1,04	1,0385	1,03	1,03	1,03
U-Theil Infla	0,9077	0,93344	0,69426	0,936	0,844	1,0041	1,0032	1,0037	1,013	0,97	1,01	1,01

The BVAR(20) specification suggested by the HQC and AIC methods results in model that outperforms clearly the traditional VAR(20) model in terms of accuracy as it can be seen in the Graph 3 whre RMSFE of both models is presented. U-Theil results show during almost of all the sample BVAR forecasts outperform traditional VARs but no in some of the monetary policy decision horizons as it is the case for h=4 and h =12 where they perform worse.

Graph 4: RMSFE BVAR(20) vs VAR(20)



Source: Own elaboration with BCP data

Table 2: U-Theil Statistic First subsample

BVAR(20)	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11	h=12
U-Theil Imae	3,05802	1,07553	0,262	0,379	0,0283	0,29	0,463	0,5128	0,55	0,72	0,37	0,25
U-Theil Int rate	0,1011	0,09042	0,033	0,147	0,2171	0,2165	0,0396	0,2096	0,27	0,32	0,3	0,3
U-Theil Int Infla	0,36443	0,23297	0,07	1,039	0,2376	0,5454	0,7626	0,5723	0,09	0,01	0,14	1,43

Forecast subsample 2

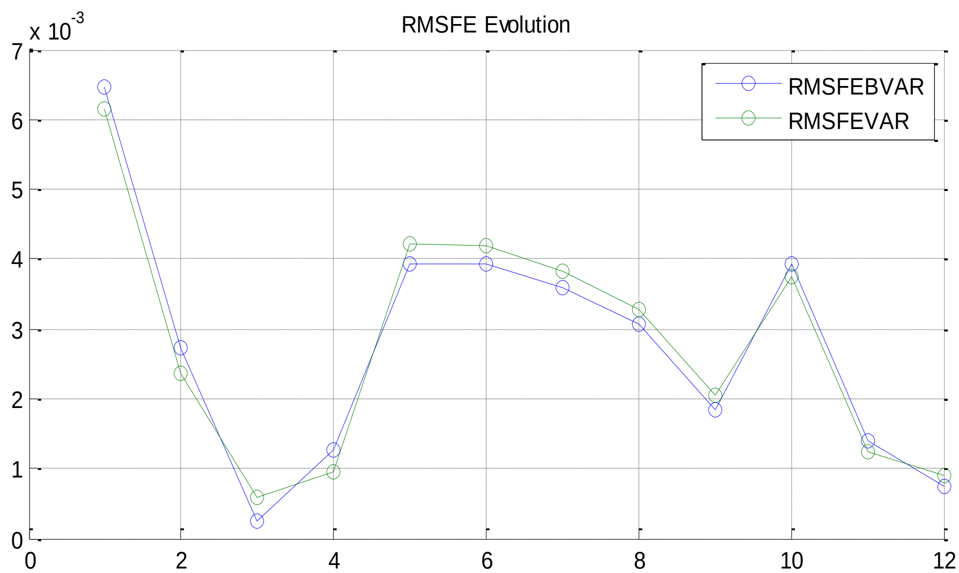
The forecast subsample ranges from 2003:01 to 2004:01. The selected models are a BVAR(23) and a BVAR(1) and as before, the compete with their homologous VAR specification. The Table VII containing the results of the U-Theil statistic for the BVAR (1) show the quality of the forecasts is better in imae rather than for the interest rate or the inflation.

Table 3: U-Theil Statistic Second subsample 1 Lag model specification

BVAR(1)	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11	h=12
U-Theil Imae	0,544012003	0,46059	0,5523	0,625	0,76	0,6476	0,7486	0,7332	0,8665	0,87	0,85	0,85
U-Theil Int rate	1,342011107	1,29201	1,29079	0,165	0,877	0,9275	0,968	0,975	0,9852	0,99	0,99	0,99
U-Theil Infla	1,054506314	1,15813	0,40838	1,337	0,932	0,9382	0,9385	0,9347	0,9048	1,05	1,13	0,83

The comparison among the competing models shows that for the case of the BVAR(1) versus the VAR(1), Bayesian approach yields a slightly more accurate forecast for the Bayesian model rather than the VAR, as it can be seen in graph 4 in which it yields the best performance from horizon 5 to 11.

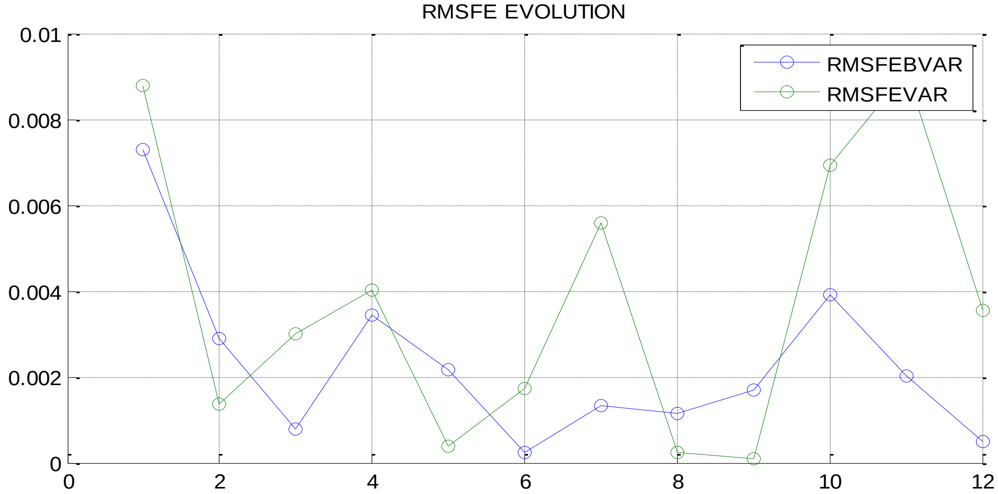
Graph 4. BVAR(1) vs VAR(1)



Source: Own elaboration with BCP data

For the case of the BVAR(23) what I find is that BVAR(23) beats in terms of accuracy the VAR(23) model. It beats traditional VAR in all classical horizons of interest for monetary policy decision making, 1step, 4steps and 12 steps ahead.

Graph 5: BVAR(23) vs VAR(23)



Source: Own elaboration with BCP data

Table 4: U-Theil Second subsample, 23 Lags model specification

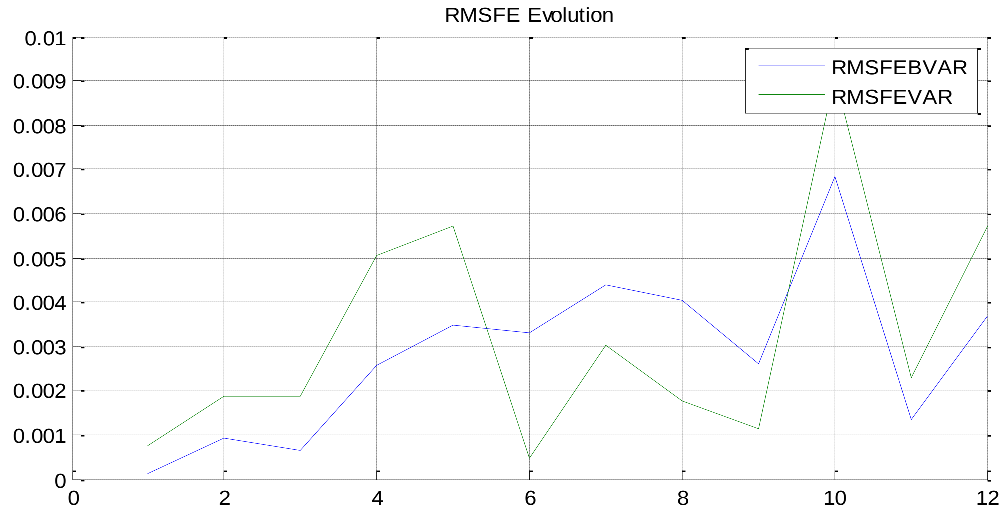
BVAR(23)	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11	h=12
U-Theil Imae	0,93379	0,11851	0,011	0,103	0,3473	1,373	0,1528	0,5419	0,5	0,5	0,32	0,82
U-Theil Int rate	0,48844	0,70882	0,234	0,082	0,6466	24,48	1,7059	1,9079	10,4	8,25	4,35	5,06
U-Theil Int Infla	0,83086	2,12967	0,26	0,856	5,7719	0,1453	0,2404	5,0926	17,3	0,57	0,22	0,14

Thus on the second subsample of check Bayesian methods outperform with both models, BVAR(1) and BVAR(23) the UVAR(p) corresponding opponents.

Forecast subsample 3

In this subsample that ranges from 2006:04:2007:04 Bayesian approach leads again, to more accurate forecasts for the short term and the long term as it can be seen in graph 5.

Graph 6. RMSFE BVAR(14) Vs VAR(14)



Source: Own elaboration with BCP data

Table 5. U-Theil Statistic third subsample

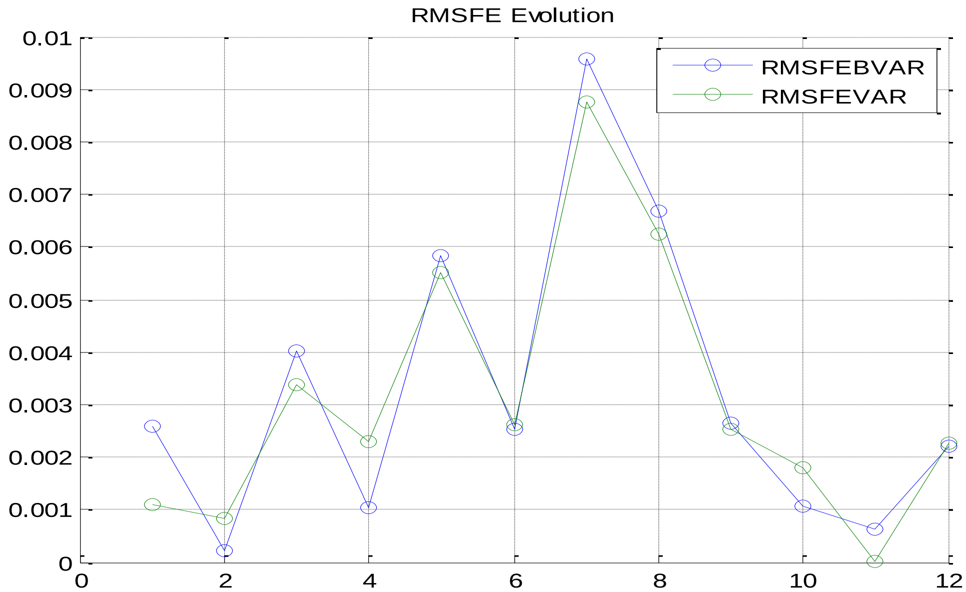
BVAR(14)	h=1	H=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11	h=12
U-Theil Imae	1,36443294	0,23704	0,69559	0,235	0,718	0,768	0,2296	0,3369	0,0044	0,34	0,36	0,41
U-Theil Int rate	1,097169909	0,97068	8,58503	0,831	0,914	0,1933	0,0035	0,1249	0,2107	0,38	0,44	0,6
U-Theil Int Infla	0,172544965	0,5057	0,35117	0,508	0,605	6,8435	1,4588	2,2901	2,297	0,75	0,59	0,65

As it shows the Table 5, Bayesian methods clearly outperform VAR since the U-Theil is usually below 1. However in this subsample, the biggest difference between BVAR and VAR in terms of performance is found in forecasting the imae. Inflation forecasts are dominated by Bayesian VAR(14) from the first period to the sixth one and again from the 9th period to the 12th. Taking the mean of the RMSFE one can observe that traditional VAR methods (0,0031) outperform BVAR (0,0033). The graphical illustration is attached in Graph 6 below.

Forecast subsample 4

In this subsample, the Bayesian approach performs poorly when compared to traditional VAR. For the first policy horizon VAR(12) delivers more accurate results accounting for a forecast 2.36 times more precise. On the fourth horizon Bayesian outperforms traditional VAR and as it is measured in table 6 for 8 and 12 months, the U-Theil statistics show that in general they are very similar. However, in overall accuracy VAR (12) is the winner in this subsample.

Graph 6: RMSFE BVAR(12) vs VAR(12)



Source: Own elaboration with BCP data

Table 6: U-Theil Statitics fourth subsample

BVAR(13)	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11	h=12
U-Theil Imae	0,74134	0,67875	3,755	1,603	0,7811	1,9093	9,8632	0,2695	0,64	0,93	0,92	0,38
U-Theil Int rate	1,29671	2,08221	1,453	1,737	3,5514	10,941	10,484	16,432	3,74	27,7	10,4	1,47
U-Theil Int Infla	2,36791	0,27159	1,189	0,444	1,0548	0,9702	1,094	1,0697	1,05	0,59	59,9	0,98

With the analysis performed above I have shown how in general, using the Minnesota prior in Bayesian time series modeling delivers more accurate forecasts than traditional VAR models for the case of forecasting monthly inflation in Paraguay. However, results should be taken cautiously since for most of the cases the differences are low and only remarkable when treating with highly parameterized structures, the priors seem to produce important effects, at least for the first three subsamples. Only in one out of the four subsamples of check I find VAR improving Bayesian approach.

7. Conclusions

The main finding of this research is that BVAR models have shown to be more accurate than traditional unrestricted VARs for the 75% of the turbulent subsamples for which coefficients have been estimated and forecasts computed. The measures implemented here to test for the model accuracy have been the RMSFE and U-Theil. Using quarterly data increases the support of the empirical evidence. The forecasts results of the period ranging from 2009Q2-2010Q1 using quarterly data with Bayesian models also show lower RMSFEs than VARs. Monetary authorities on the light of this research can have a good ally by using this forecast method.

8. References

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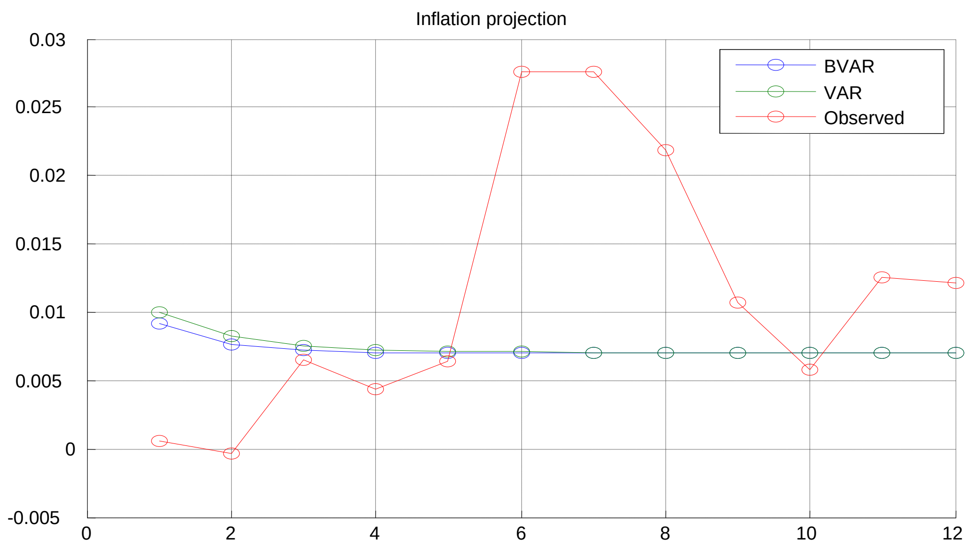
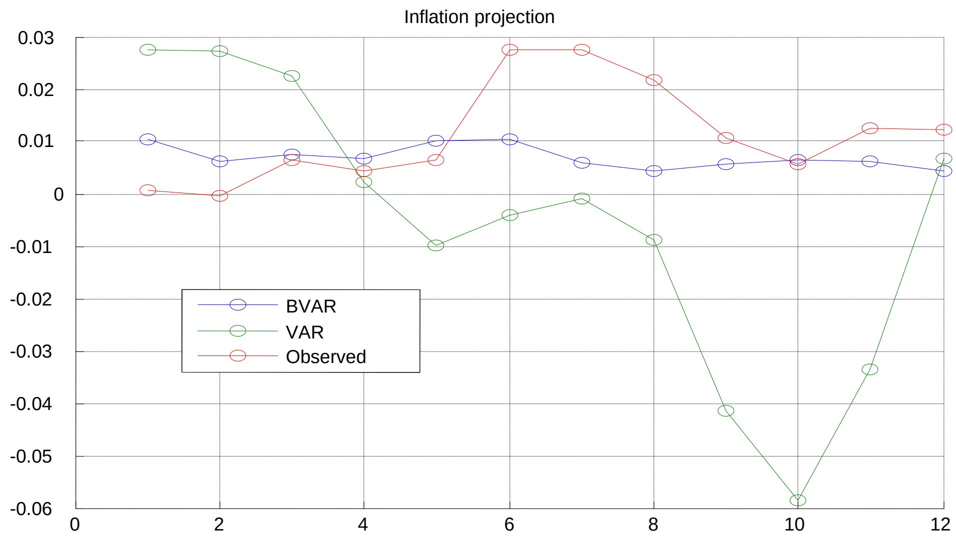
9. Appendix

9.1 Monthly forecasts

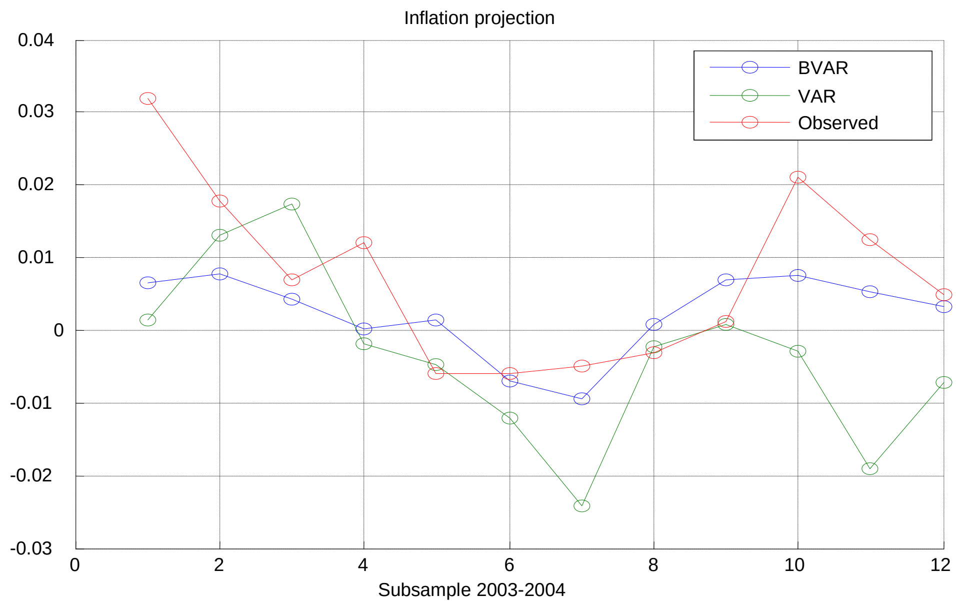
First Forecast period: 2002-2003, BVAR(1)



First Forecast period: 2002-2003, BVAR(1), BVAR(20)

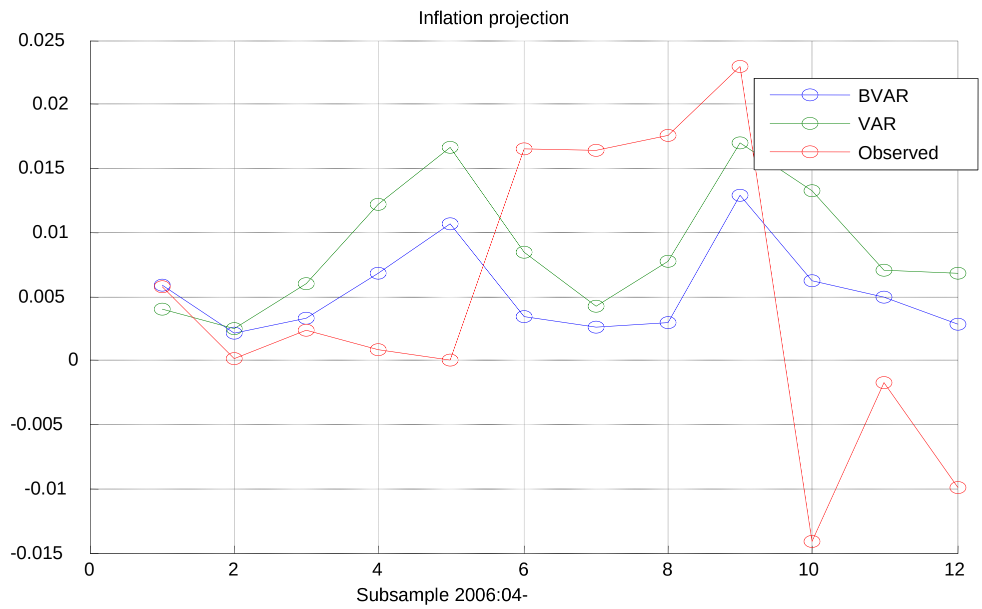


Second Forecast period: 2003-2004, BVAR(23)



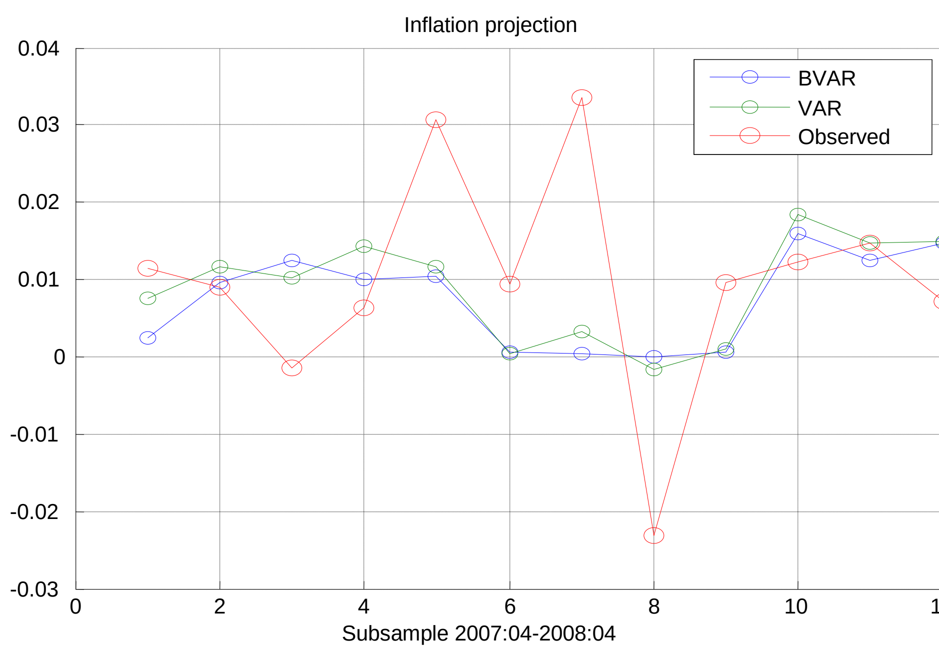
Third Forecast period: 2006:04-2007:04

BVAR (14)



Fourth Forecast period. 2007: 05-2008:05

BVAR(12)



9.2 Tables: Information Criterias

Estimation subsample 1: 1996-2002

Table 10: Model selection criteria statistics

Retardo	s log.veros	p(RV)	AIC	BIC	HQC
1	416,44633		-12,444503	-12,043077*	-12,286115
2	426,16066	0,02179	-12,466482	-11,763987	-12,189303
3	430,99765	0,37752	-12,338389	-11,334826	-11,942419
4	436,97921	0,2154	-12,245514	-10,940882	-11,730753
5	438,45209	0,96641	-12,01391	-10,408209	-11,380358
6	453,21479	0,00053	-12,191224	-10,284454	-11,438881
7	458,2227	0,34921	-12,068391	-9,860551	-11,197257
8	463,86656	0,2565	-11,965125	-9,456216	-10,9752
9	471,14812	0,10366	-11,91225	-9,102272	-10,803534
10	485,49544	0,00073	-12,076783	-8,965736	-10,849276
11	494,1601	0,0438	-12,066465	-8,654349	-10,720166
12	548,57099	0	-13,463723	-9,750538	-11,998633
13	570,69386	0	-13,867503	-9,85325	-12,283623
14	581,03316	0,01416	-13,908712	-9,59339	-12,206041
15	596,27145	0,00036	-14,10066	-9,484269	-12,279198
16	611,48259	0,00037	-14,291772	-9,374312	-12,351519
17	622,20031	0,01085	-14,344625	-9,126095	-12,28558
18	667,14598	0	-15,450646	-9,931047	-13,27281
19	715,69146	0	-16,66743	-10,846762	-14,370803
20	764,87795	0	**-17,903937***	-11,7822	**-15,488519***

Table 11: Estimation subsample 2: 1996-2003

Retardo	s log.veros	p(RV)	AIC	BIC	HQC
1	470,03613		-12,379355	**-12,005723***	-12,230308
2	478,63928	0,04558	-12,368629	-11,714773	-12,107798
3	482,95303	0,47234	-12,241974	-11,307893	-11,869358
4	488,19962	0,31205	-12,14053	-10,926226	-11,656129
5	490,3513	0,89034	-11,955441	-10,460912	-11,359255
6	504,30681	0,00099	-12,089373	-10,31462	-11,381402
7	511,44199	0,11303	-12,038973	-9,983996	-11,219217
8	519,07231	0,08402	-12,001954	-9,666753	-11,070414
9	527,15765	0,0634	-11,977234	-9,361809	-10,933909
10	541,42691	0,00077	-12,119646	-9,223997	-10,964536
11	551,20319	0,02088	-12,140627	-8,964753	-10,873732
12	599,40257	0	-13,20007	-9,743972	-11,82139
13	608,08942	0,04318	-13,191606	-9,455284	-11,701141
14	627,24936	0,00002	-13,466199	-9,449653	-11,863949
15	634,25441	0,12197	-13,412281	-9,115511	-11,698247
16	651,64237	0,00007	-13,638983	-9,061989	-11,813164
17	661,42346	0,02082	-13,660093	-8,802875	-11,722489
18	674,12942	0,00255	-13,760255	-8,622812	-11,710866
19	695,22332	0	-14,087117	-8,66945	-11,925943
20	740,63164	0	-15,071125	-9,373235	-12,798167
21	755,40257	0,00052	-15,227096	-9,248982	-12,842353
22	793,27198	0	-16,007351	-9,749012	-13,510823
23	892,36444	0	-**18,442282***	-11,903719	**-15,833969***

Table 12: Estimation subsample 3: 1996-2006:05

Lags	log.veros	p(RV)	AIC	BIC	HQC
1	685,1318	NaN	-13,622636	**-13,51843**	**-13,580462**
2	685,28244	1,39586	-13,585648	-13,429338	-13,522388
3	686,48288	0,54646	-13,569658	-13,361244	-13,48531
4	686,53398	1,64234	-13,53068	-13,270162	-13,425244
5	686,54214	1,856	-13,490842	-13,178222	-13,36432
6	687,0288	0,97084	-13,460576	-13,095852	-13,312966
7	687,35742	1,13296	-13,427148	-13,010322	-13,25845
8	687,46744	1,48022	-13,389348	-12,920418	-13,199564
9	687,51886	1,64122	-13,350378	-12,829344	-13,139506
10	688,66378	0,56924	-13,333276	-12,760138	-13,101316
11	692,25316	0,1163	-13,365064	-12,739822	-13,112018
12	707,34732	0,0002	-13,626946	-12,949602	-13,352814
13	708,36782	0,6248	-13,607356	-12,877908	-13,312136
14	714,01172	0,03504	**-13,680234**	-12,898684	-13,363926
15	714,01228	1,96224	-13,640246	-12,806592	-13,30285
16	714,69978	0,81404	-13,613996	-12,728238	-13,255514
17	714,71964	1,77584	-13,574392	-12,636532	-13,194824
18	715,01392	1,17498	-13,540278	-12,550314	-13,139622
19	715,03928	1,74694	-13,500786	-12,458718	-13,079042
20	721,23562	0,0256	-13,584712	-12,49054	-13,141882
21	721,333	1,50998	-13,54666	-12,400386	-13,082742
22	721,7837	1,004	-13,515674	-12,317296	-13,030668
23	722,5849	0,74146	-13,491698	-12,241216	-12,985606
24	726,71824	0,0841	-13,534364	-12,23178	-13,007186

Table 13: Estimation subsample 4: 1996-2007:05

Lags	log.veros	p(RV)	AIC	BIC	HQC
1	-580,6222	0	8,826602	**9,33813**	9,034474
2	-569,74274	0,56812	8,93055	9,825728	9,294328
3	-559,70702	0,6952	9,046818	10,325642	9,5665
4	-532,41506	0,0025	8,911168	10,57364	9,586756
5	-502,59764	0,00094	8,738652	10,78477	9,570144
6	-492,59152	0,69998	8,85535	11,285116	9,842748
7	-469,18596	0,0107	8,776438	11,58985	9,91974
8	-458,6986	0,62498	8,88611	12,08317	10,185318
9	-451,37436	1,2068	9,041962	12,622668	10,497074
10	-442,68706	0,93324	9,177914	13,142266	10,78893
11	-407,51752	1,20E-04	8,927264	13,275264	10,694184
12	-254,82234	0	6,96091	11,692558	**8,883736**
13	-237,53968	0,08894	6,971382	12,086678	9,050114
14	-230,91	1,35122	7,137372	12,636314	9,372008
15	-224,76572	1,4508	7,310448	13,193038	9,70099
16	-192,40712	0,00034	7,100834	13,36707	9,64728
17	-179,55938	0,33924	7,17605	13,825932	9,8784
18	-156,47936	0,01204	7,101888	14,13542	9,960144
19	-143,86482	0,36168	7,180508	14,597686	10,19467
20	-123,70216	0,03386	7,148936	14,949762	10,319002
21	-111,87316	0,44624	7,239024	15,423496	10,564994
22	-106,76442	1,64946	7,427218	15,995338	10,909094
23	-79,7769	0,00282	7,296014	16,24778	10,933794
24	-27,37272	0	**6,793762**	16,129176	10,587448

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