RISING MIDDLE CLASS AND VENDOR FINANCING INCENTIVES IN UNSECURED CREDIT MARKETS IN LATIN AMERICA

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Documentos de Trabajo

N° 20

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This Draft: June 2017

Abstract

I illustrate through the lens of a theoretical model, potential reasons triggering an increase in credit supplied by the non traditional financial sector, i.e vendors, at the extensive margin. I find that a reduction in the average risk of default and an increase in the market size of credit customers raise vendor financing incentives. This model rationalizes the observation that the improvement of economic conditions of the low-income and financially constrained households potentially led to increased credit supply by vendors in several countries of Latin America.

JEL Classification: E21, E49, G2

JEL Keywords: Consumer credit, Unsecured Debt, Endogenous Financial Contracts, Vendor Financing, Latin America

1 Introduction

During the recent decade, as Latin America have been experiencing a rise in the middle class population (Ferreira et al., 2013), consumer credit markets in the region were also observing an increase in credit from the non traditional financial sector. In particular, in countries such as Mexico, Colombia, Chile and Brazil, international and national level retail chains emerged as the main credit suppliers of the lower-middle income population (Obermann (2006), Ruiz-Tagle et al. (2013) and Montero and Tarzijan (2010)).

According to Casanova and Renck (2015), in spite of a significant decline in unemployment in recent years, the lack of formal employment and poor credit history were still impeding

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many individuals from gaining access to consumer finance from traditional financial institutions. In order to allow "new middle class shoppers" access non-essential items typically offered by large retail stores, the retailers themselves started offering credit.

Motivated by the emergence of vendors as consumption credit suppliers in these Latin American countries, I set up a theoretical model of vendor financing in the unsecured credit market. The model illustrates the fundamentals affecting vendor financing incentives that could increase credit supply on its extensive margin.

There are two strands of literature which we build on and contribute to.

The first strand is the study of consumption credit and default. Much of this literature focuses on explaining stylized facts of the US credit market related to the evolution of bankruptcy filing and consumption credit. There is indeed active research trying to explain why the personal bankruptcy rate in the US has increased more than threefold in the last two decades. Since there is an increasing consensus that the rise in bankruptcies is primarily driven by consumer debt market developments particularly related to IT progress, most of this literature is gravitating towards the study of this link. Another theme in this literature is what Livshits et al. (2016) call *democratization of credit* and what Drozd and Serrano-Padial (2016) call *revolving revolution*, i.e., the extension of credit to new and seemingly riskier borrowers in the recent decades. This rise in credit on the extensive margin is driven by financial innovations in the former work and IT adoption by the debt collection industry in the latter work. It may also arise naturally in different models[1]. We extend the literature on unsecured credit by including an alternative type of lender -vendors- whose business model differs from that of banks. This exercise is highly relevant given that vendors are an important source of consumption credit in developing countries[2].

The second strand is the research related to trade finance. One of the earliest papers with a stylized model of vendor financing incentives is by Brennan et al. (1988). Their model suggests that one reason why retailers find profitable to extend credit is that customers differ in their price elasticity and vendor financing is a channel enabling them to price discriminate and increase overall sales. However, the type of credit is secured - the good purchased using credit is also used as the collateral. Not surprisingly, subsequent papers both on the empirical and theoretical front, evolved towards studying inter-firm credit. To my knowledge, there is no theoretical work studying manufacturer incentives to provide unsecured credit to the final good consumer. This research aims to be a first step to fill that gap.

[1] See Livshits (2015) for a review of papers in this literature.

[2] Livshits (2015) argued that one key challenge that he doesnâĂŹt think has been successfully addressed yet is modelling a consumer credit market where borrowers may deal with multiple lenders

In this paper, we present a stylized model of vendor financing in an unsecured credit market, following the intuition by Brennan et al. (1988). Vendors face two types of customers - those who have unlimited access to credit and are able to buy their good using cash or bank credit (cash customers), and those financially constrained with low cash in hand that need credit to purchase the good (credit customers). We study potential reasons triggering an increase in the credit supplied by the non traditional financial sector, i.e vendors, at the extensive margin. We particularly focus on the effect of a reduction in the average risk of default and the market size of credit customers on vendor financing incentives. In addition, we also examine the effect of certain structural parameters in the model - one reflecting financial development and the other capturing bankruptcy costs.

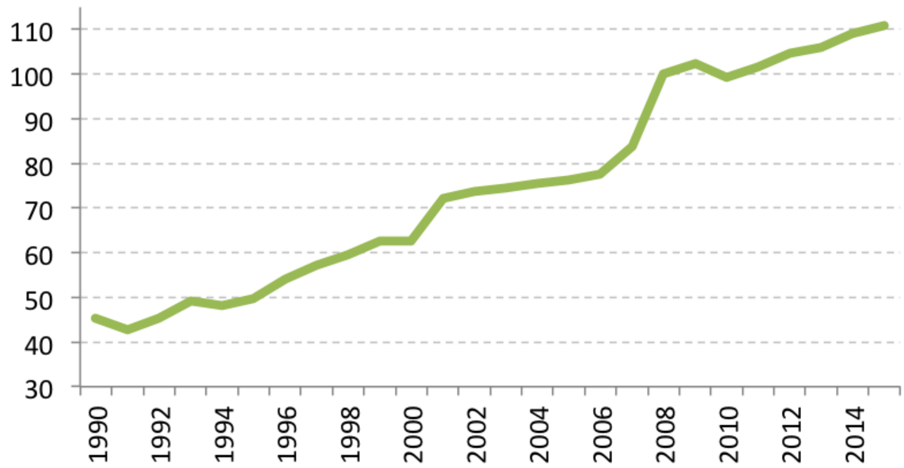
This paper is structured as follows. Section 2 describes the development of commercial stores as non traditional consumption credit suppliers in Chile. Section 3 outlines the theoretical model of vendor financing in the unsecured credit market. Section 4 presents the analytical results of the vendor's optimization problem with and without vendor financing. Section 5 enumerates cases depending on structural parameters of the model and the corresponding vendor financing gains function. Section 6 derives the core results of the paper through comparative statics exercises. Section 7 provides preliminary empirical evidence supporting the model's main mechanism and an important model implication.

2 Understanding the rise of vendor financing using Chile as case study

Using the Chilean case as a research motivation is sensible given that it was Chile where the vendor financing business model flourished, and later expanded regionally. Even though Brazilian retailers pioneered the offer of installment payment plans, Chilean retailers provide the earliest and most successful stories of offering store cards (Calderón Hoffmann, 2006). Once the Chilean market came close to saturation, Chilean retail conglomerates expanded to other Latin American countries through acquisitions of local chains or local operations of multinational retailers[3].

We start by looking at aggregate measures of credit market depth in Chile. According to a report by the SBIF (2015), debt to income ratio (DIR) experienced a significant increase in the last two decades, jumping from 35% in 2001 to 61% in 2015. Similarly, domestic credit to private sector as % of GDP increased from 45.3% in 1990 to 110.9% in 2015 and observed a clear upward trend during the selected period as seen in figure 1.

Figure 1: Domestic credit to private sector in Chile (% GDP)



Source: World Bank - World Development Indicators (WDI)

The greatest change that occurred in the Chilean consumption credit market during the period 2000-2008 is the significant increase in non-bank debt (Central Bank of Chile (2009)). Figure 2 illustrates the evolution of credit cards per adult in Chile and shows that commercial stores were the most dynamic participant in this market during the period 1993-2007. The number of active credit cards per adult provided by commercial stores increased significantly after 1995

[3]The second big surge of investment abroad from Chile occurred in 2003 and it was lead by retail companies (i.e Cencosud , Falabella, Ripley). See Calderón Hoffmann (2006).

and stayed well above those provided by banks throughout the period 1995-2007.

Figure 3 illustrates the evolution of total credit supplied by commercial stores in Chile, expressed in constant prices. We can infer from the figure that consumers' outstanding debt with commercial stores grew faster than real GDP during the period 2000-2008.

There is a lack of precise information regarding when exactly these new participants in the credit market became more apparent. Marshall (2004) points out that while in the early 90s, consumption credit in Chile was mainly supplied by the traditional banking sector, new suppliers of financial services to households emerged in the late 90s.

According to Aparici and Yáñez (2004), as banks were decreasing their participation in total consumption debt during the period 1999-2003, commercial stores were placing themselves as second most important source of consumption credit. Figure 2 provides evidence that the relevance of commercial stores in the market of credit cards became more visible in the early 2000s.

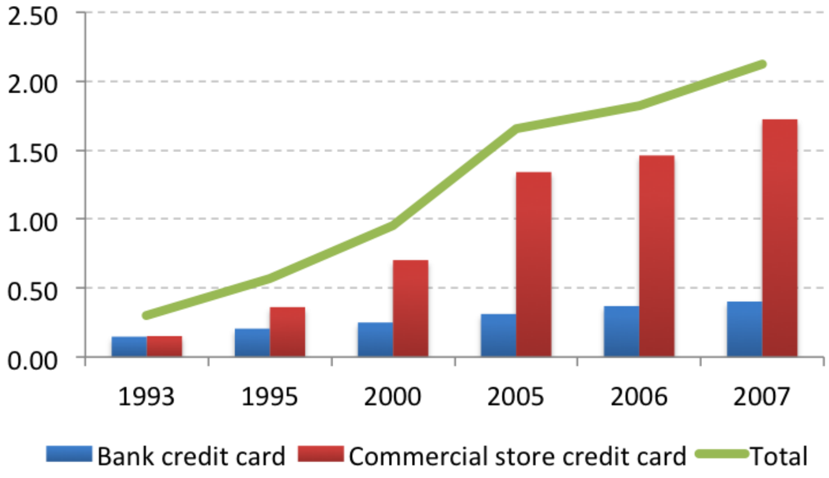
In addition to the emergence of these new credit suppliers, the significant rise in consumption credit was also a consequence of the increase in credit demand by new sections of the population (SBIF, 2015). Indeed, Chile was the country with the highest middle class population growth within the Latin American region during the period 1995-2010 (Ferreira et al., 2013). About 20% of the population was considered middle class in 1995 and this percentage jumped to 53% by 2010. In figure 4, I plot the shifting composition of the population in Chile across income class. While the upper and vulnerable classes remained relatively stable, there is a clear downward trend for the poor households, and a significant rise in the middle class.

Casanova and Renck (2015) explain how an increase in the consumer market size driven by the rising middle class and a *delayed response by banks* led retailers to offer credit themselves, in order to boost sales and increase profits.

There are other proposed explanations of the rise of vendors as a non-traditional source of credit. One is related to marketing strategies. In particular, the provision and use of store credit cards, which mainly serve purchasing within the stores of its affiliates, improves customer retention rates (Samsing, 2011).

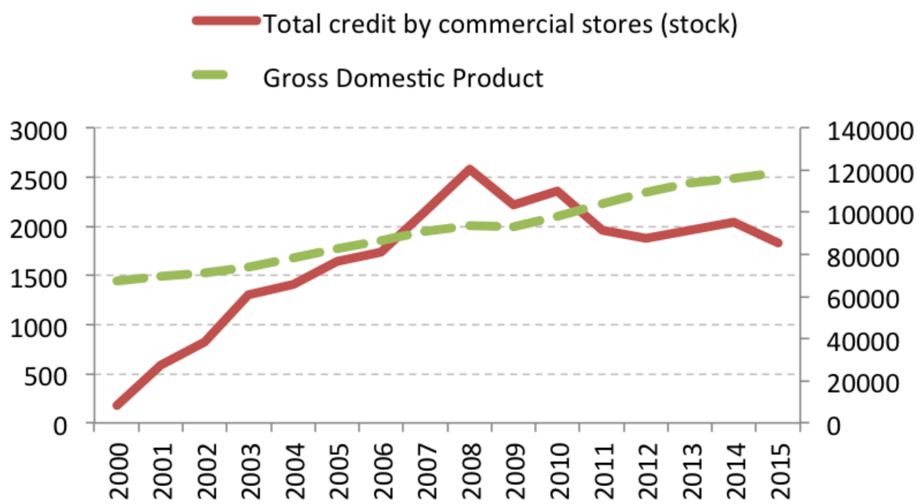
Another explanation is a change in the regulatory framework implemented in 1999 by the SBIF, the authority responsible for monitoring and regulating the financial market in Chile. The new regulation led to the *segmentation* of the interest rate ceiling. This regulation increased the maximum rate of interest that financial and non-financial lenders (including commercial stores) could charge borrowers, particularly when provided credit in small amounts. Many specialists claimed that this significantly stimulated the supply of credit cards (Rojas (2011), Benado

Figure 2: Bank and non bank credit cards (active) per adult in Chile



Source: Superintendencia de Bancos e Instituciones Financieras de Chile

Figure 3: The evolution of total credit by commercial stores and GDP in Chile (thousand millions of 2011 chilean pesos)



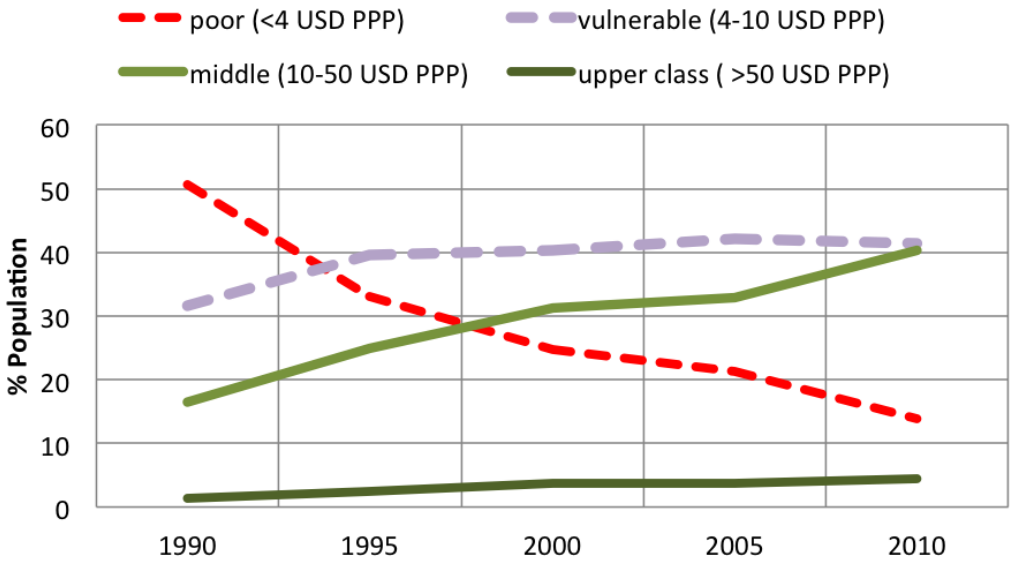
Source: Superintendencia Valores y Seguros Chile (SVS) and World Bank - World Development Indicators (WDI)
Notes: Total credit by commercial stores reflects stock of outstanding consumption debt with vendors, including refinanced loans. Nominal values were converted to constant prices by diving the series with the implicit GDP deflator- extracted from the OECD database.

(2011)).

Finally, a scandal involving a particular Chilean commercial store in 2011[4] led policy makers to start questioning this sector's lending practices, and their increasing role in the consumer credit market. There was widespread public attention to the matter - see Barrionuevo (2011), Knowledge@Wharton (2011) and Evans (2014) - and studies of vendor financing incentives from

[4]A recount of the accounting scandal involving the retailer La Polar can be found in McMillan (2012)

Figure 4: Percentage of total households by income class in Chile (5 year mean)



Source: Author calculations based on database by Ferreira et al. (2013)
Notes: The classification of income class has been determined for Latin America by the World Bank and is expressed in 2005 US$ PPP (purchasing power parity)

a theoretical perspective were called upon. In the next section, we present a first attempt to model vendor financing incentives in the market for unsecured credit.

3 Introducing vendor financing in the unsecured credit market

The model presented in this paper adapts the stylized model of Brennan et al. (1988) and uses it to shed light on the plausible factors behind the rise of vendor financing in Latin America. We modify their model on the credit demand and credit supply setup. We substitute farmers demanding credit with households maximizing utility from consumption, and accumulating durable good services through purchases of vendor's goods. In our model, both banks and vendors' captive financial intermediary offer unsecured credit contracts.

There are five agents in the model- a competitive bank sector, a profit maximizing vendor, the vendor's captive financial intermediary and two types of households (constrained and unconstrained).

3.1 Households

3.1.1 Constrained households

Constrained households derive utility from their consumption in non-durables (c_t) and services from durables (d_t). However, since they don't have enough cash in hand to purchase durable goods, they need access to a source of finance to do so. A **key assumption** in this model is that if constrained households receive a credit offer from banks, they use it solely to finance the purchase of one unit of the durable good, commercialized at price z_1 set by the vendor.

There are two periods. The household's labor income in the two periods is denoted by y_1 and y_2. The first period income is pre-determined and consists of household's cash in hand, i.e his labor income net of debt repayment. The second period income, y_2, is stochastic taking one of two possible values $y_2 \in \{y_L, y_H\}$. Households differ in the probability ρ of receiving the high income y_H. We identify households with type ρ where $\rho \sim Beta(\alpha, \beta)$. Borrowing households know their type.

In the two period optimization problem, we assume each household chooses non-durable consumption for two periods (c_1, c_2) and on receiving a credit offer, decide whether to accept it and purchase one unit of durable good in the first period or not.

We assume CRRA preferences over a CES aggregator of non-durable consumption and services from durable goods. Consistent with empirical findings[5], we assume that period utility

[5] Fernández-Villaverde and Krueger (2011) reviews previous empirical literature estimating CRRA utility functions with a CES aggregator and using US consumption data. Findings suggest that the intratemporal elasticity of substitution -between services flows from durables and nondurables- is not significantly different from one.

takes the Cobb-Douglas form:

$$U(c_t, d_t) = \frac{\left(c_t^{\gamma} d_t^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} \tag{1}$$

where σ measures the degree of risk aversion and γ captures the weight of each type of consumption in household preferences ($0 < \gamma < 1$).

If the household hasn't received a credit offer, then they won't purchase any durable goods and they face the following two period optimization problem:

$$\begin{aligned} \max_{\{c_1^i, c_2^i, \textit{Purchase or No Purchase}\}} \quad & U(c_1^i, d_1^i) + \beta U(c_2^i, d_2^i) \\ & \textit{subject to:} \\ & c_1^i \leq y_1^i \\ & c_2^i \leq y_2^i \\ & d_1^i = (1-\delta) d_0^i \\ & d_2^i = (1-\delta)^2 d_0^i \end{aligned}$$

If the household receives a credit offer, they must choose whether to accept or reject it. If rejected, then the solution of the previous problem applies. If accepted, then household proceeds to purchase one unit of durable good. In this case, $d_1^i = 1 + (1-\delta)d_0^i$ and $d_2^i = (1-\delta) + (1-\delta)^2 d_0^i$

The value of accepting a loan will factor in the possibility of default. With probability ρ, the household receives high income y_H in period 2 and pays back the loan repayment value z_2. Conversely, with probability $1-\rho$ they receive low income y_L and default. Following the literature of unsecured credit, if they default, they suffer a utility cost which is equivalent to losing share ϕ of second period income. Regardless of paying back or not, they still hold the durable good purchased in period 1.

The value of autarky (i.e not buying durable goods) for the household is:

$$V^{nb}(d_0, \rho) = U(y_1, (1-\delta)d_0) + \beta\rho U(y_H, (1-\delta)^2 d_0) + \beta(1-\rho) U(y_L, (1-\delta)^2 d_0) \tag{2}$$

The value of accepting the credit offer (or equivalently the value of buying one unit of durable good) is:

$$\begin{aligned} V^b(d_0, \rho, z_2) = & U(y_1, 1 + (1-\delta)d_0) + \beta\rho U(y_H - z_2, (1-\delta) + (1-\delta)^2 d_0) + \ldots \\ & \beta(1-\rho) U((1-\phi)y_L, (1-\delta) + (1-\delta)^2 d_0) \end{aligned}$$

Then a household will accept the credit offer and purchase one unit of durable good as long as

$$V^b(d_0, \rho, z_2) \geq V^{nb}(d_0, \rho)$$

For simplicity, we **assume** that constrained households have such low durable good stock (d_0^c) that for any $z_2 \leq \phi y_H$, the value of accepting the credit offer and purchase the good is always larger than the value of autarky.

That is, given the share of income lost if default (ϕ), preference parameters (γ,β) and depreciation rate δ, their durable good stock satisfies:

$$\frac{(1-\phi)^{\frac{\beta\gamma}{(1+\beta)(1-\gamma)}}}{1-(1-\phi)^{\frac{\beta\gamma}{(1+\beta)(1-\gamma)}}} \geq d_0(1-\delta)$$

Derivations in Appendix 8.1.

3.1.2 Unconstrained households

Unconstrained households derive utility from their consumption in non-durables (c_t) and services from durables (d_t). However, they don't need access to credit to increase their expenditure in durable goods since they have unlimited access to credit provided by banks. Preferences are symmetric to those of constrained households.

As in Brennan et al. (1988), these households default with probability 0 in their credit contract. This implies banks offer them credit loans at rate equal to the risk free interest rate. We assume that unconstrained households not only have first period income net of debt payments greater than that of constrained households but also no uncertainty regarding their second period income. This is aligned with data suggesting households with financial inclusion tend to have higher income, more assets and overall lower default risk. For simplicity, we assume there is no income heterogenity among unconstrained households.

In the two period optimization problem, the unconstrained households choose non-durable consumption for two periods (c_1, c_2) and decide to purchase one unit of durable good in the first period or not. They face the following maximization problem:

$$\begin{aligned} \max_{\{c_1, c_2, \textit{Purchase or No Purchase}\}} \quad & U(c_1, d_1) + \beta U(c_2, d_2) \\ \textit{subject to:} \quad & c_1 + z_1 x_1 + \frac{c_2}{R_1^B} = y_1 + \frac{y_2}{R_1^B} \\ & d_1 = x_1 + (1-\delta)d_0 \end{aligned} \tag{3}$$

where z_1 is the relative price of durable goods and $x_1 \in \{0, 1\}$ stands for units of durable goods purchased. Remember we assume that if a household decides to purchase durable goods, they can only buy one unit per period.

The first order condition for c_1 yields:

$$\begin{aligned} U_1(c_1^*, d_1) =& \beta U_1(c_2^*, d_2) R_1^B \\ & \textit{with: } c_2^* = R_1^B(y_1 - c_1^* - z_1 x_1) + y_2 \end{aligned}$$

If the household finds purchasing the durable good optimal, $x_1 = 1$ and durable services for the first and second period are $d_1 = 1 + (1-\delta)d_0$ and $d_2 = (1-\delta) + (1-\delta)^2 d_0$, respectively. The first order condition for c_1 yields:

$$\begin{aligned} U_1(c_1^{p*}, 1 + (1-\delta)d_0) =& \beta U_1(c_2^{p*}, (1-\delta) + (1-\delta)^2 d_0) R_1^B \\ & \textit{with: } c_2^{p*} = R_1^B(y_1 - c_1^{p*} - z_1) + y_2 \end{aligned}$$

If household finds not purchasing the durable good optimal, $x_1 = 0$ and durable services for the first and second period are $d_1 = (1-\delta)d_0$ and $d_2 = (1-\delta)^2 d_0$, respectively. Then, first order condition for c_1 yields:

$$\begin{aligned} U_1(c_1^{np*}, (1-\delta)d_0) =& \beta U_1(c_2^{np*}, (1-\delta)^2 d_0) R_1^B \\ & \textit{with: } c_2^{np*} = R_1^B(y_1 - c_1^{np*}) + y_2 \end{aligned}$$

To determine the decision of buying the durable good or not, the household compares the value of buying (V_r^b) versus the value of not buying (V_r^{nb}):

$$\begin{aligned} V_r^b =& U(c_1^{p*}, 1 + (1-\delta)d_0) + \beta U(R_1^B(y_1 - c_1^{p*} - z_1) + y_2, (1-\delta) + (1-\delta)^2 d_0) \\ V_r^{nb} =& U(c_1^{np*}, (1-\delta)d_0) + \beta U(R_1^B(y_1 - c_1^{np*}) + y_2, (1-\delta)^2 d_0)) \end{aligned}$$

An unconstrained household will choose to purchase one unit of durable good as long as:

$$V_r^b \geq V_r^{nb}$$

Notice the value of purchase is negatively related to the relative price of durable goods z_1 and the value of no purchase is independent of z_1. This will guarantee that there is a unique intersection (z_1^*) of both value functions such that for values of $z_1 < z_1^*$, it is optimal to purchase the good, ceteris paribus.

Let $y_1 = y_2 = \bar{y}$, then

$$\frac{(1 + R_1^B)}{R_1^B} \bar{y} \;\; \Omega(d_0, \delta, \gamma) = z_1^* \tag{4}$$

where

$$\Omega(d_0, \delta, \gamma) = 1 - \left(\frac{1}{(1-\delta)d_0} + 1 \right)^{-\frac{(1-\gamma)}{\gamma}}$$

Note, the lower their durable good stock (d_0) or the higher their income ($\bar{y}$), the higher is the maximum cash price (z_1^*) at which they accept to purchase the durable good.

3.2 Banks

This section builds on the profit function of banks described in Livshits et al. (2016)[6].

Banks are competitive, they borrow at the exogenously given gross interest rate R^F and make loans to borrowers. Loans take the form of one period non-contingent bond contracts. However, to offer a new contract, financial intermediaries incur in a fixed cost χ.

The fixed cost to create a lending contract represents the cost of developing a screening technology (i.e scorecards), which allows the lender to perfectly assess borrower's risk types[7]. Thus, upon paying the fixed cost χ, a lender observes borrower's type. Since each prospective borrower is infinitesimal relative to this fixed cost, lending contracts have to pool the different constrained household types to recover the cost of creating the contract.

The contract posted is characterized by $(z1, R^B, \underline{\rho})$, where R^B is the gross interest rate and $\underline{\rho}$ is the probability of repayment cut-off defining which households are eligible. The amount advanced in period 1 is denoted by z_1 and is equivalent to the cash price of durable goods set optimally by the vendor.

Since the eligibility decision is made after the fixed cost has been incurred, lenders are willing to accept any household who yields non negative operating profits. In other words, the riskiest household accepted makes no contribution to the overhead cost χ. Hence a lender offering a risky loan at interest rate R^B rejects all applicants with risk type below a cutoff $\underline{\rho}$ such that the expected return from the marginal borrower is zero: $\frac{\underline{\rho} z_2}{R^F} - z_1 = 0$, where z_2 $(= z_1 \times R^B)$ is the repayment value. The marginal type accepted into the contract is

$$\underline{\rho} = \frac{R^F}{R^B} \tag{5}$$

The profit to the lender of extending the credit contract $(z1, R^B, \underline{\rho})$ to constrained households is:

$$\Pi = -\chi + \int_{\underline{\rho}}^{1} \left(\frac{\rho R^B}{R^F} - 1 \right) z_1 \times f(\rho) d\rho$$

where $f(\rho)$ is the probability density function evaluated at ρ. Note the upper limit of the integral is set at 1. This follows from the assumption that unconstrained households have such low durable good stock that -regardless of their risk profile- the value of accepting the credit offer and purchase the good is greater than the value of remaining in autarky.

Since banks are perfectly competitive, profits in equilibrium are zero. In equilibrium, and

[6] See section 7 for a brief explanation on why we choose to differ from the stylized bank described in Brennan et al. (1988)

[7] We assume perfect information

after substituting R^B using equation (5), we get:

$$\chi = \int_{\underline{\rho}}^{1} \left(\frac{\rho}{\underline{\rho}} - 1\right) z_1 \times f(\rho) d\rho \tag{6}$$

Since the right hand side of equation (6) is decreasing in $\underline{\rho}$, there will be a unique $\underline{\rho}$ for each z_1, given χ, R^F and the distribution of ρ. All households with $\rho \geq \underline{\rho}$ are offered (and accept) this contract.

In the section describing unconstrained households, we stated they have unlimited access to credit provided by banks and that they have zero probability of default. This can be rationalized by the existence of a lender offering a one period bond contract only to these households and with an interest rate equal to the risk free interest rate. In this setup with fixed costs, this lender would have zero fixed costs (i.e $\chi = 0$).

Recall from section 3.1.1, that if these households receive high income in second period ($y_2 = y_H$), they won't default on their debt. This implies repayment value (z_2) should be lower or equal than the income lost if household defaults (ϕy_H). This condition defines an upper bound z_1^c such that for all z_1 higher than that value, banks are not able to extend credit to constrained households since all borrowers will default with certainty.

In particular, the value z_1^c solves:

$$z_1^c = \phi y_H \underline{\rho}^c$$

where $\underline{\rho}^c$ is derived from:

$$\chi = \int_{\underline{\rho}^c}^{1} (\rho - \underline{\rho}^c) \phi y_H \times f(\rho) d\rho \tag{7}$$

Note $\underline{\rho}^c$ is a function of ϕ, y_H, the distribution of $\rho \sim Beta(\alpha, \beta)$ and fixed costs χ.

At the same time, there will be a value z_1^{min} at which for all z_1 lower than that value, the corresponding $\underline{\rho}$ derived from 6 yields an interest rate higher than the ceiling rate policy (R^{max}). We will **assume** hereafter that $\boldsymbol{z_1^{min} < z_1^c}$. If this assumption doesn't hold, then neither banks nor vendors have incentives to pay the fixed cost and offer a new credit contract since all borrowers will default at the implied repayment value.

Define the region $z_1 \in [z_1^{min}, z_1^c]$ as the feasible set over which banks extends credit to constrained consumers. Then, the corresponding total number of risky borrowers is defined as:

$$q(z_1) = \begin{cases} 0 & \text{\textit{If} } z_1 < z_1^{min} \\ N^c \times (1 - G(\underline{\rho})) & \text{\textit{If} } z_1 \in [z_1^{min}, z_1^c] \\ 0 & \text{\textit{If} } z_1 > z_1^c \end{cases} \tag{8}$$

where N^c is the total number of constrained households and $G(.)$ is the cumulative distribution function of risk types.

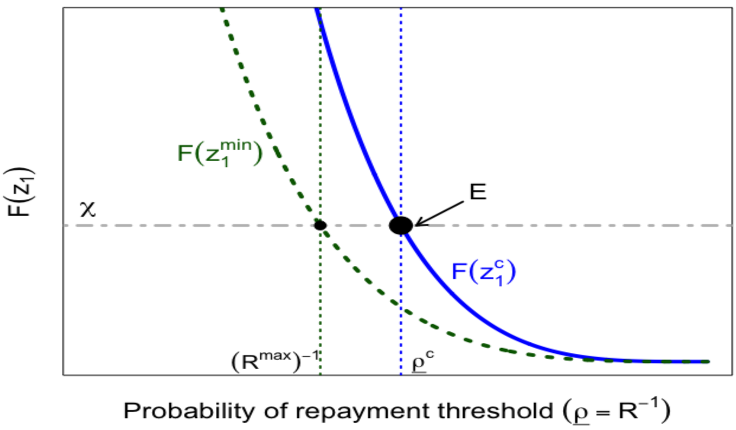
Figure 5 illustrates the bank problem. Figure 5a illustrates the corresponding cutoff of probability of repayment at z_1^{min} and z_1^c. The dashed and continuous lines evaluate the right hand side of equation 6 at z_1^{min} and z_1^c, respectively. The intersection of the continuous line with the fixed cost given by the horizontal line determines the probability of repayment of the marginal borrower and therefore the interest rate of the contract.

Figure 5b shows the repayment value z_2 as an increasing function of the amount advanced in period 1 (z_1). See proof in Appendix 8.2. Note that the feasible region $[z_1^{min}, z_1^c]$ will also yield a lower and upper bound for the repayment value represented in the y-axis.

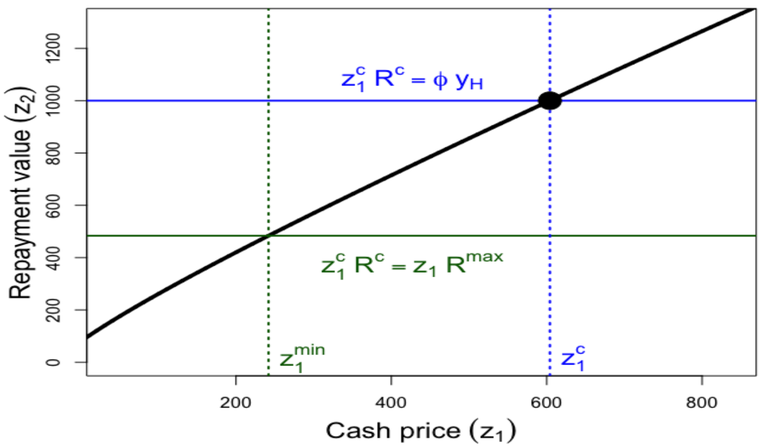
Figure 5c represents number of risky borrowers as function of the cash price. It is zero for low values, peaks at z_1^{min} and is a decreasing function of the cash price up to z_1^c.

Figure 5: The bank problem

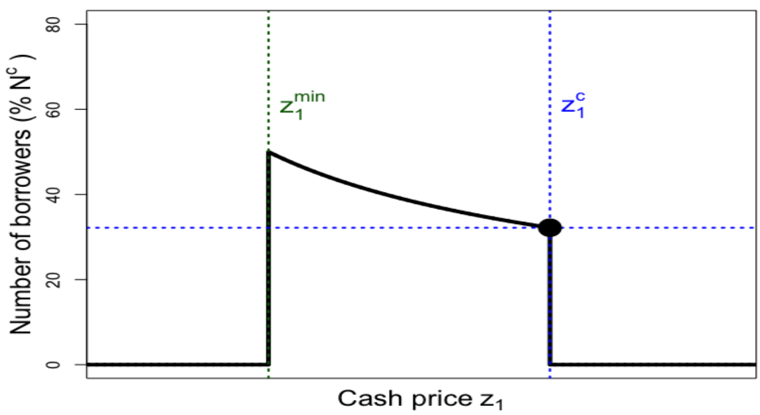
(a) Deriving $\underline{\rho}$ given z_1, fixed costs χ and distribution $\rho \sim Beta(\alpha, \beta)$



(b) Repayment value (z_2) in equilibrium



(c) Credit consumers (% of total constrained) in equilibrium



Value z_1^c satisfies $z_1^c = \phi y_H \underline{\rho}^c$ where $\underline{\rho}^c$ is financial intermediary probability of repayment threshold at z_1^c. Value z_1^{min} corresponds to value of cash price for which financial intermediary's probability threshold corresponds to the inverse of ceiling rate (R^{max}). All subfigures assume $\chi = 40$, $\phi = 0.1$, $yh = 10000$, $R^{max} = 2$, $E(\rho) = 0.5$ and $\sigma^2(\rho) = 3.5\%$.

3.3 The vendor

Consider a company that sells consumption goods to households. The goods that are produced at a constant marginal cost ν, may be purchased by the households either using cash or credit. Credit can be offered by the competitive banking system -previously described- or by the vendor itself (through its captive finance subsidiary).

Remember there are two types of consumers: constrained households and unconstrained households. The former are considered "credit customers" by the vendor since they require access to credit to purchase durable goods they sell. The latter are considered "cash customers" since they have unlimited access to a risk free credit market and can pay using cash. As in Brennan et al. (1988), vendors find profitable to extend credit to constrained customers because they differ in their price elasticity relative to cash customers. By doing so it enables them to price discriminate and increase overall sales.

Absence of vendor financing

The vendor chooses the cash price z_1 that maximizes profits. As it will be presented in the next section, under certain conditions, the optimal z_1 may attract both credit and cash customers.

The problem of the manufacturer in the absence of vendor financing is:

$$\max_{z_1} \Pi(C) = N_r(z_1 - v) + q(z_1)(z_1 - v)$$

$$\textit{subject to:}$$

$$V_r^b(z_1) \geq V_r^{nb}$$

where N^r is the number of cash customers and q is the number of credit customers defined in the Bank section. The constraint guarantees that cash customer buys the good. That is the value of purchasing the good for cash customers is equal or greater than the value of no purchase.

With vendor financing

The problem of the manufacturer who offers vendor financing is to figure out two prices. In addition to the price offered to cash customers (z_1), they need to solve for the internal transfer price at which goods are sold to a captive finance subsidiary. That is, the manufacturer is able to charge a lower price to constrained households by setting the internal transfer price (z_1') below the cash market price.

The competitive captive financial subsidiary faces the same fixed costs χ, same gross interest rate R^F at which they borrow and same optimization problem relative to banks. The only difference is that they observe a lower cash price. That is, the cutoff $\underline{\rho}$ solves:

$$\chi = \int_{\underline{\rho}}^{1} \left(\frac{\rho}{\underline{\rho}} - 1\right) z_1' \times f(\rho)d\rho \tag{9}$$

As with the Bank, the total number of risky borrowers that results depends on the value of z_1' :

$$q(z_1', \chi) = \begin{cases} 0 & \textit{If } z_1' < z_1^{min} \\ N^c \times (1 - G(\underline{\rho})) & \textit{If } z_1' \in [z_1^{min}, z_1^c] \\ 0 & \textit{If } z_1' > z_1^c \end{cases} \tag{10}$$

With vendor financing, the manufacturer's profit maximization problem is:

$$\max_{z_1, z_1'} \Pi(z_1, z_1') = N_r(z_1 - v) + q(z_1', \chi)(z_1' - v) \tag{11}$$

Subject to:

$$V_r^b(z_1) \geq V_r^{nb}(z_1) \tag{11a}$$

$$z_1' R^V \geq z_1 R^F \tag{11b}$$

$$z_1' \leq z_1 \tag{11c}$$

$$z_1' R^V \leq \phi y_H \tag{11d}$$

Contraint (11a) is a condition guaranteeing that cash customer buys the good. This sets the upper bound for the cash price z_1.

Constraint (11b) guarantees that no cash customer buys on credit provided by the vendor. This constraint ensures that the present value of the quoted credit price ($z_1' R^V / R^F$) is not less than the cash price z_1. Derivations in Appendix 8.3.

Constraint (11c) implies banks won't be able to offer a better credit contract to credit customers than vendors. By subsidizing the amount advanced in period 1, the captive financial intermediary is able to charge a lower repayment value z_2 to credit customers[8].

Finally, the last constraint (11d) defines an upper bound for the internal transfer price z_1'. All values above this upper bound imply that its corresponding repayment value is greater than the cost of default and all borrowers choose to default. By setting this constraint, the vendor ensures that the captive financial intermediary is able to extend credit to constrained households.

[8] The repayment value is an increasing function of z_1. Proof in Appendix 8.2

4 Solving vendor's optimization problem

4.1 Absence of vendor financing

The problem of the manufacturer in the absence of vendor financing is:

$$\begin{aligned} \max_{z_1} \Pi(z_1) =& N_r(z_1 - v) + q(z_1)(z_1 - v) \\ &\textit{subject to:} \end{aligned} \tag{12}$$

$$z_1^{max}(R^F, \bar{y}, d_0^u) \geq z_1 \tag{12a}$$

where $z_1^{max}(R^F, \bar{y}, d_0^u)$ is the maximum cash price at which unconstrained consumers purchases the vendor's good

$$z_1^{max}(R^F, \bar{y}, d_0^u) = \frac{(1 + R^F)}{R^F}\bar{y}\ \ \Omega(d_0^u, \delta, \gamma)$$

$$\Omega(d_0^u, \delta, \gamma) = 1 - \left(\frac{1}{(1-\delta)d_0^u} + 1\right)^{-\frac{(1-\gamma)}{\gamma}}$$

Parameters $\bar{y}$ and d_0^u are income per period and durable good stock of unconstrained consumers, respectively. See derivation of z_1^{max} in Appendix 8.5.2.

Remember the number of customers purchasing the good using credit not only depends on the credit customer market size (N^c) but also on the distribution of probability of repayment and the cash price set by the vendor. The cash price set by the vendor influences the interest rate and the number of constrained consumers receiving a credit offer. If it surpasses a given ceiling z_1^c then no constrained consumer can pay back and no credit contract is offered. If it is lower than z_1^{min}, the equilibrium interest rate is larger than that allowed by the ceiling rate policy and no credit is offered.

$$q(z_1) = \begin{cases} 0 & \textit{If } z_1 < z_1^{min} \\ N^c \times (1 - G(\underline{\rho})) & \textit{If } z_1 \in [z_1^{min}, z_1^c] \\ 0 & \textit{If } z_1 > z_1^c \end{cases}$$

Ceiling z_1^c is the maximum amount advanced in period 1 at which credit customer is able to pay back in period 2 when they receive income y_H. That is z_1^c is the maximum z_1 that satisfies $y_H - z_1 \times R^B \geq y_H - \phi y_H$. In simpler terms, $z_1^c R^B = \phi y_H$ where ϕy_H is the amount of high income lost if consumer defaults on its debt. The value z_1^c solves

$$\chi = \int_{z_1^c/(\phi y_H)}^{\bar{a}} \left(\frac{\rho}{z_1^c/(\phi y_H)} - 1\right) z_1^c \times f(\rho) d\rho$$

Next we present the optimal solution by case.

4.1.1 Case I: $z_1^c < z_1^{max}$

Since $q(z_1)$ takes three functional forms depending on the value of z_1, we define three Lagrangians for the vendor's optimization problem

For $\mathbf{z_1 > z_1^c}$,

$$\mathcal{L}(z_1, \lambda) = N_r(z_1 - v) + \lambda[z_1^{max} - z_1]$$

The corresponding Kuhn-Tucker conditions

$$\begin{aligned}
\mathcal{L}_1(z_1^*, \lambda^*) =& N_r - \lambda^* = 0 \\
\mathcal{L}_2(z_1^*, \lambda^*) =& [z_1^{max} - z_1^*] \geq 0 \\
& \lambda^* \geq 0 \\
& \lambda[z_1^{max} - z_1^*] = 0
\end{aligned}$$

These conditions are satisfied when:

$$z_1^* = z_1^{max} \qquad \lambda^* = N_r$$

For $\mathbf{z_1^{min} < z_1 \leq z_1^c}$,

$$\mathcal{L}(z_1, \lambda) = N_r(z_1 - v) + q(z_1)(z_1 - v) + \lambda[z_1^c - z_1]$$

The corresponding Kuhn-Tucker conditions

$$\begin{aligned}
\mathcal{L}_1(z_1^*, \lambda^*) =& N_r + q(z_1^*) + (z_1^* - v)\frac{dq(z_1)}{dz_1}\bigg|_{z_1 = z_1^*} - \lambda^* = 0 \\
\mathcal{L}_2(z_1^*, \lambda^*) =& [z_1^c - z_1^*] \geq 0 \\
& \lambda^* \geq 0 \\
& \lambda[z_1^c - z_1^*] = 0
\end{aligned}$$

These conditions are satisfied when:

$$z_1^* = z_1^c \qquad \lambda^* = N_r + q(z_1^*) + (z_1^* - v)\frac{dq(z_1)}{dz_1}\bigg|_{z_1 = z_1^*} > 0$$

$$Note \quad \lambda^* > 0 \quad since \quad \frac{dq(z)}{dz_1} > 0 \quad \forall z_1 \in [v, z_1^{max}]$$

Result $\lambda^* > 0$, follows from $\frac{d(q(z_1) \times (z_1 - v))}{dz_1} > 0$. Derivation in Appendix 8.4.

For $\mathbf{z_1 \leq z_1^{min}}$, the lagrangian at the optimum is

$$\mathcal{L}(z_1^*, \lambda^*) = N_r(z_1^* - v) + q(z_1^*)(z^* - v) + \lambda[z_1^{min} - z_1]$$

where Kuhn-Tucker conditions are defined similarly as above and are satisfied when:

$$z_1^* = z_1^{min} \qquad \lambda^* = N_r + q(z_1^*) + (z_1^* - v)\frac{dq(z_1)}{dz_1}\bigg|_{z_1=z_1^*}$$

Given these three solutions, we define three profit functions:

$$\begin{aligned}
\Pi_0^{I,A} =& N_r(z_1^{max} - v) \\
\Pi_0^{I,B} =& N_r(z_1^c - v) + q(z_1^c)(z_1^c - v) \\
\Pi_0^{I,C} =& N_r(z_1^{min} - v) + q(z_1^{min})(z_1^{min} - v)
\end{aligned}$$

Note $\Pi_0^{I,C} < \Pi_0^{I,B}$ $\forall v$ and N_r. Therefore, the optimal choice z_1^* can be summarized as follows:

$$z_1^* = \begin{cases} z_1^{max} & \textit{If} \quad \frac{z_1^{max}-v}{z_1^c-v} \geq 1 + \frac{N^c}{N^r}\left(1 - G(\underline{\rho}(z_1^c))\right) \\ \\ z_1^c & \textit{If} \quad \frac{z_1^{max}-v}{z_1^c-v} < 1 + \frac{N^c}{N^r}\left(1 - G(\underline{\rho}(z_1^c))\right) \end{cases}$$

The corresponding number of credit customers at each solution

$$q(z_1^*, \chi) = \begin{cases} 0 & \textit{If } z_1^* = z_1^{max} \\ N^c\left(1 - G(\underline{\rho}(z_1^c))\right) & \textit{If } z_1^* = z_1^c \end{cases}$$

The corresponding profit functions at each solution

$$\Pi_I^{NVF} = \begin{cases} N_r(z_1^{max} - v) & \textit{If} \quad \frac{z_1^{max}-v}{z_1^c-v} \geq 1 + \frac{N^c}{N^r}\left(1 - G(\underline{\rho}(z_1^c))\right) \\ \\ N_r(z_1^c - v) + q(z_1^c, \chi)(z_1^c - v) & \textit{If} \quad \frac{z_1^{max}-v}{z_1^c-v} < 1 + \frac{N^c}{N^r}\left(1 - G(\underline{\rho}(z_1^c))\right) \end{cases}$$

Note that the solution $z_1^* = z_1^c$ is more likely when the number of credit customers $q(z_1^c)$ is significantly higher than that of cash consumers (N^r). Another set of conditions under which z_1^c may be the optimal solution is when z_1^{max} decreases and moves closer to z_1^c. In other words, when second period income for the cash customer $\bar{y}$ decreases and/or their durable good stock is high enough.

4.1.2 Case II: $z_1^c \geq z_1^{\boldsymbol{max}}$

Assuming that is always optimal to sell to cash customers, there are two subcases.

If $\mathbf{z_1^{max}} > \mathbf{z_1^{min}}$,

$$\mathcal{L}(z_1,\lambda) = N_r(z_1 - v) + q(z_1,\chi)(z_1 - v) + \lambda[z_1^{max} - z_1]$$

The corresponding Kuhn-Tucker conditions

$$\begin{aligned}
\mathcal{L}_1(z_1^*,\lambda^*) =& N_r + q(z_1^*,\chi) + (z_1^* - v)\frac{dq(z_1)}{dz_1}\bigg|_{z_1=z_1^*} - \lambda^* = 0 \\
\mathcal{L}_2(z_1^*,\lambda^*) =& [z_1^{max} - z_1^*] \geq 0 \\
& \lambda^* \geq 0 \\
& \lambda[z_1^{max} - z_1^*] = 0
\end{aligned}$$

These conditions are satisfied when:

$$z_1^* = z_1^{max} \qquad \lambda^* = N_r + q(z_1^{max},\chi) + (z_1^{max} - v)\frac{dq(z_1,\chi)}{dz_1}\bigg|_{z_1=z_1^{max}}$$

If $\mathbf{z_1^{max}} < \mathbf{z_1^{min}}$

$$\mathcal{L}(z_1,\lambda) = N_r(z_1 - v) + \lambda[z_1^{max} - z_1]$$

The corresponding Kuhn-Tucker conditions

$$\begin{aligned}
\mathcal{L}_1(z_1^*,\lambda^*) =& N_r - \lambda^* = 0 \\
\mathcal{L}_2(z_1^*,\lambda^*) =& [z_1^{max} - z_1^*] \geq 0 \\
& \lambda^* \geq 0 \\
& \lambda[z_1^{max} - z_1^*] = 0
\end{aligned}$$

These conditions are satisfied when:

$$z_1^* = z_1^{max} \qquad \lambda^* = N_r$$

Note that it is always optimal to set $\boldsymbol{z_1^* = z_1^{max}}$. The corresponding number of credit customers:

$$q(z_1^{max}) = \begin{cases} 0 & \textit{If } z_1^{max} < z_1^{min} \\ N^c\left(1 - G(\varrho(z_1^{max}))\right) & \textit{If } z_1^{max} \geq z_1^{min} \end{cases}$$

The corresponding profit:

$$\Pi_{II}^{NVF} = \begin{cases} N_r(z_1^{max} - v) & \textit{If } z_1^{max} < z_1^{min} \\ \\ N_r(z_1^{max} - v) + q(z_1^{max})(z_1^{max} - v) & \textit{If } z_1^{max} \geq z_1^{min} \end{cases}$$

4.2 With vendor financing

With vendor financing, the manufacturer's profit maximization problem is:

$$\begin{aligned} \max_{z_1, z_1'} \Pi(z_1, z_1') =& N_r(z_1 - v) + q(z_1', \chi)(z_1' - v) \\ & \textit{subject to:} \end{aligned} \tag{13}$$

$$z_1^{max}(R^F, \bar{y}, d_0^u) \geq z_1 \tag{13a}$$

$$\frac{z_1'}{\varrho(z_1')} \geq z_1 R^F \tag{13b}$$

$$z_1' \leq z_1 \tag{13c}$$

$$\frac{z_1'}{\varrho(z_1')} \leq \phi y_H \tag{13d}$$

where

$$z_1^{max}(R^F, \bar{y}, d_0^u) = \frac{(1 + R^F)}{R^F} \bar{y} \times \Omega(d_0^u, \delta, \gamma)$$

$$\Omega(d_0^u, \delta, \gamma) = 1 - \left(\frac{1}{(1 - \delta) d_0^u} + 1 \right)^{-\frac{(1-\gamma)}{\gamma}}$$

Next we present the optimal solution by case.

4.2.1 Case I: $z_1^c < z_1^{max}$

We define the Lagrangian as

$$\begin{aligned} \mathcal{L}(z_1, z_1', \lambda_1, \lambda_2, \lambda_3, \lambda_4) =& N_r(z_1 - v) + q(z_1')(z_1' - v) + \\ & + \lambda_1 [z_1^{max} - z_1] + \lambda_2 \left[\frac{z_1'}{\varrho(z_1')} - z_1 R^F \right] + \\ & + \lambda_3 [z_1 - z_1'] + \lambda_4 \left[\phi y_H - \frac{z_1'}{\varrho(z_1')} \right] \end{aligned}$$

The corresponding Kuhn-Tucker conditions

$$
\begin{aligned}
\mathcal{L}_1(z_1^*, z_1'^*, \boldsymbol{\lambda^*}) =& N_r - \lambda_1^* - \lambda_2^* R^F + \lambda_3^* = 0 \\
\mathcal{L}_2(z_1^*, z_1'^*, \boldsymbol{\lambda^*}) =& q(z_1'^*) + (z_1'^* - v)\frac{dq(z_1)}{dz_1}\bigg|_{z_1=z_1'^*} + \lambda_2^* \left[\frac{1}{\rho(z_1)} - \frac{z_1}{\rho(z_1)^2}\frac{d\rho}{dz_1}\right]\bigg|_{z_1=z_1'^*} + \dots \\
& - \lambda_3^* - \lambda_4^* \left[\frac{1}{\rho(z_1)} - \frac{z_1}{\rho(z_1)^2}\frac{d\rho}{dz_1}\right]\bigg|_{z_1=z_1'^*} = 0 \\
\frac{d\mathcal{L}(z_1^*, z_1'^*, \boldsymbol{\lambda^*})}{d\lambda_1} =& z_1^{max} - z_1 \geq 0 \\
\frac{d\mathcal{L}(z_1^*, z_1'^*, \boldsymbol{\lambda^*})}{d\lambda_2} =& \frac{z_1'}{\rho(z_1')} - z_1 R^F \geq 0 \\
\frac{d\mathcal{L}(z_1^*, z_1'^*, \boldsymbol{\lambda^*})}{d\lambda_3} =& z_1 - z_1' \geq 0 \\
\frac{d\mathcal{L}(z_1^*, z_1'^*, \boldsymbol{\lambda^*})}{d\lambda_4} =& \phi y_H - \frac{z_1'}{\rho(z_1')} \geq 0
\end{aligned}
$$

$$
\begin{aligned}
& \lambda_1^* \geq 0 \;\; \lambda_2^* \geq 0 \;\; \lambda_3^* \geq 0 \;\; \lambda_4^* \geq 0 \\
& \lambda_1^*[z_1^{max} - z_1] = 0 \\
& \lambda_2^* \left[\frac{z_1'}{\rho(z_1')} - z_1 R^F\right] = 0 \\
& \lambda_3^*[z_1 - z_1'] = 0 \\
& \lambda_4^* \left[\phi y_H - \frac{z_1'}{\rho(z_1')}\right] = 0
\end{aligned}
$$

We have two cases,

a) If $R^F z_1^{max} \leq \phi y_H$

The K-T conditions are satisfied at

$$
\begin{aligned}
& z_1^* = z_1^{max} \qquad\qquad z_1'^* = z_1^c \\
& \lambda_1^* = N^r; \;\; \lambda_2^* = 0; \;\; \lambda_3^* = 0 \\
& \lambda_4^* = \left(q(z_1^c) + (z_1^c - v)\frac{dq(z_1)}{dz_1}\bigg|_{z_1=z_1^c} \right) \times \left(\left[\frac{1}{\rho(z_1)} - \frac{z_1}{\rho(z_1)^2}\frac{d\rho}{dz_1}\right]^{-1}\bigg|_{z_1=z_1^c} \right) > 0
\end{aligned}
$$

b) If $R^F z_1^{max} > \phi y_H$

The K-T conditions are satisfied at

$$z_1^* = \frac{\phi y_H}{R^F} \qquad\qquad z_1'^* = z_1^c$$
$$\lambda_1^* = 0; \;\; \lambda_2^* = \frac{N^r}{R^F}; \;\; \lambda_3^* = 0$$
$$\lambda_4^* = \left(q(z_1^c) + (z_1^c - v)\frac{dq(z_1)}{dz_1}\bigg|_{z_1=z_1^c} \right) \times \left(\left[\frac{1}{\varrho(z_1)} - \frac{z_1}{\varrho(z_1)^2}\frac{d\varrho}{dz_1} \right]^{-1} \bigg|_{z_1=z_1^c} \right) + \frac{N^r}{R^F} > 0$$

Summarizing, the optimal choice $(z_1^*, z_1'^*)$ can be described as follows:

$$(z_1^*, z_1'^*) = \begin{cases} (z_1^{max}, z_1^c) & \textit{If} \quad R^F z_1^{max} \leq \phi y_H \\ \\ (\frac{\phi y_H}{R^F}, z_1^c) & \textit{If} \quad R^F z_1^{max} > \phi y_H \end{cases}$$

Note that it is always optimal to set the internal transfer price z_1' at the maximum z_1^c, since the first derivative of the profit function relative to z_1' is always positive, regardless of the mean and variance of probability of repayment distribution[9].

The profit function for each corresponding case:

$$\Pi_I^{VF} = \begin{cases} N^r(z_1^{max} - v) + q(z_1^c, \chi)(z_1^c - v) & \textit{If} \quad R^F z_1^{max} \leq \phi y_H \\ \\ N^r\left(\frac{\phi y_H}{R^F} - v\right) + q(z_1^c, \chi)(z_1^c - v) & \textit{If} \quad R^F z_1^{max} > \phi y_H \end{cases}$$

4.2.2 Case II: $z_1^c \geq z_1^{max}$

We define the Lagrangian as

$$\begin{aligned} \mathcal{L}(z_1, z_1', \lambda_1, \lambda_2, \lambda_3, \lambda_4) =& N_r(z_1 - v) + q(z_1', \chi)(z_1' - v) + \\ & + \lambda_1[z_1^{max} - z_1] + \lambda_2\left[\frac{z_1'}{\varrho(z_1')} - z_1 R^F\right] + \\ & + \lambda_3[z_1 - z_1'] + \lambda_4\left[\phi y_H - \frac{z_1'}{\varrho(z_1')}\right] \end{aligned}$$

In this case, it is always optimal to set $\boldsymbol{z_1^* = z_1'^* = z_1^{max}}$. That means price discrimination doesn't increase profits. The corresponding number of credit customers:

$$q(z_1^{max}, \chi) = \begin{cases} 0 & \textit{If } z_1^{max} < z_1^{min} \\ N^c\left(1 - G(\varrho(z_1^{max}))\right) & \textit{If } z_1^{max} \geq z_1^{min} \end{cases}$$

[9] This result follows from $\frac{d(q(z_1)\times(z_1-v))}{dz_1} > 0$, proved in Appendix.

The corresponding profit:

$$\Pi_{II}^{VF} = \begin{cases} N_r(z_1^{max} - v) & \textit{If } z_1^{max} < z_1^{min} \\ \\ N_r(z_1^{max} - v) + q(z_1^{max}, \chi)(z_1^{max} - v) & \textit{If } z_1^{max} \geq z_1^{min} \end{cases}$$

5 Vendor financing gains

To derive vendor financing gains, we will only focus in the case $z_1^c < z_1^{max}$. The previous section showed that if this condition doesn't hold, then there are no incentives to use vendor financing as a mean to price discriminate customers.

Given all structural parameters in the economy, we define vendor financing gains (VF) as the difference between profits with vendor financing at the optimal choices of z_1 and z_1' and Profits in the absence of vendor financing at the optimal cash price z_1. There are four cases:

Case 1. $\frac{z_1^{max}-v}{z_1^c-v} \geq 1 + \frac{N^c}{N^r}\left(1 - G(\varrho(z_1^c))\right)$ *and* $R^F z_1^{max} \leq \phi y_H$

$$VF^{(1)} = q(z_1^c, \chi)(z_1^c - v) \tag{14}$$

Case 2. $\frac{z_1^{max}-v}{z_1^c-v} \geq 1 + \frac{N^c}{N^r}\left(1 - G(\varrho(z_1^c))\right)$ *and* $R^F z_1^{max} > \phi y_H$

$$VF^{(2)} = N^r\left(\frac{\phi y_H}{R^F} - z_1^{max}\right) + q(z_1^c, \chi)(z_1^c - v) \tag{15}$$

Note $VF^{(2)} > 0$ *if*

$$\frac{N^c}{N^r}\left(1 - G(\varrho(z_1^c))\right)(z_1^c - v) > z_1^{max} - \frac{\phi y_H}{R^F}$$

Case 3. $\frac{z_1^{max}-v}{z_1^c-v} < 1 + \frac{N^c}{N^r}\left(1 - G(\varrho(z_1^c))\right)$ *and* $R^F z_1^{max} \leq \phi y_H$

$$VF^{(3)} = N^r(z_1^{max} - z_1^c) \tag{16}$$

Case 4. $\frac{z_1^{max}-v}{z_1^c-v} < 1 + \frac{N^c}{N^r}\left(1 - G(\varrho(z_1^c))\right)$ *and* $R^F z_1^{max} > \phi y_H$

$$VF^{(4)} = N^r\left(\frac{\phi y_H}{R^F} - z_1^c\right) \tag{17}$$

Cases 1 and 2 correspond to the scenario at which the vendor, absent of vendor financing, finds profitable to sell only to cash customers. In cases 3 and 4 the vendor, absent of vendor financing, optimally chooses a lower cash price so that she sells to both cash and constrained consumers (the latter through bank credit).

6 Comparative statics

The core results of this section are derived from comparative statics exercises on gains from vendor financing- defined as the difference between profit of the manufacturer with vendor financing and in the absence of it.

We will evaluate how gains from vendor financing changes when

1. There is an increase in the market size of credit customers N^c.

2. There is a rise in the mean of repayment probability in the credit customer market.

3. There is a change in the fixed cost χ incurred by the financial sector (proxy for financial development).

4. There is a change in default costs (proxy for a change in bankruptcy policy)

We won't present comparative statics on vendor financing gains under cases 3 and 4 described in the previous section. These cases reflect a scenario in which a rise of vendor financing gains, ceteris paribus, is not associated with an increase in credit supply on its extensive margin but with a switch of source of credit for the constrained household sector. Instead, section 2 presents evidence that there was indeed an increase in consumption credit in Chile during the recent decade and that it was particularly driven by the emergence of vendors as new credit suppliers. Furthermore, there is evidence that this led to a rise in credit on its extensive margin as loans from vendors tend to be held by new "middle class" shoppers who need credit to purchase their goods (Casanova and Renck, 2015).

By focusing on vendor financing gains derived for cases 1 and 2, we are more aligned with data. Under both of these cases, a rise in vendor financing is correlated with greater credit access for constrained households.

6.1 An increase in the credit customer market size N^c

The following equation illustrates the partial derivative of VF with respect to the credit customer market size N^c.

$$\frac{dVF}{dN^c} = (1 - G(\varrho))\,(\phi y_H \varrho - v) > 0 \tag{18}$$

As the number of credit customers increase, vendor financing gains unambigously rise.

6.2 An increase in the mean of the distribution of ρ

Equation below illustrates the partial derivative of VF with respect to the mean μ.

$$\frac{dVF}{d\mu} = \frac{dq(N^c, \varrho)}{d\mu} \times (\phi y_H \varrho - v) + q(N^c, \chi, \phi, \mu, \sigma^2) \times \phi y_H \frac{d\varrho}{d\mu} \tag{19}$$

where

$$\frac{dq(N^c, \varrho)}{d\mu} = N^c \int_{\varrho}^{1} \frac{df(\rho)}{d\mu} d\rho - N^c f(\varrho) \frac{d\varrho}{d\mu} \tag{20}$$

$f(\rho)$ is the density function, $\frac{df(\rho)}{d\mu}$ its derivative relative to the mean and $\frac{d\varrho}{d\mu} > 0$ (see proof in Appendix 8.6.1).

A rise in the mean on the probability of repayment from constrained households implies that a higher mass of credit customers have now a lower risk of default. The pooling of borrowers to cover the same fixed costs stops at a marginal borrower relatively safer than previously. This means that the probability of repayment cutoff above which constrained households are offered a credit offer is higher. Equivalently, the interest rate offered in the contract is lower. Figure 6a illustrates the higher probability of repayment threshold after an increase in the mean.

We proved earlier that the vendor finds optimal to set the internal transfer price at its largest feasible value, i.e that at which its corresponding repayment value equals the cost of defaulting. Since there is no change in the default cost and the increase in mean decreases the interest rate, the internal transfer price will have to increase. This will tend to increase vendor financing gains. Figure 6b illustrates the higher internal transfer price on the x-axis after an increase in the mean.

At the same time, given assumptions of a relatively dispersed probability of repayment distribution (i.e variance=1.97%), a higher mean will also result in a larger number of credit borrowers. Therefore, a rise in the mean will unambiguously increase vendor financing gains. Figure 6c illustrates this result for a variance=3.97%.

However, it is worth noting that under sufficiently low variances, a higher mean will decrease the number of borrowers. But even in this case, we may still derive a condition under which vendor financing gains may still increase. In particular, gains will increase after a rise in the average probability of repayment as long as the marginal cost is higher than a threshold $v^{*\mu}$ defined as

$$\left(\varrho + \frac{\left(\int_{\varrho}^{1} f(\rho) d\rho \right) \times \int_{\varrho}^{1} (\rho - \varrho) \times \frac{df(\rho)}{d\mu} d\rho}{\left(\int_{\varrho}^{1} f(\rho) d\rho \int_{\varrho}^{1} \frac{df(\rho)}{d\mu} d\rho + f(\varrho) \int_{\varrho}^{1} (\rho - \varrho) \times \frac{df(\rho)}{d\mu} d\rho \right)} \right) \phi y_H = v^{\mu *} \tag{21}$$

See its derivation in Appendix 8.7.1.

6.3 A rise in financial intermediary's fixed costs

The partial derivative of VF with respect to fixed costs χ is defined as:

$$\frac{dVF}{d\chi} = \frac{dq(N^c, \chi, \phi, \mu, \sigma^2)}{d\chi} \times (\phi y_H \varrho - v) + q(N^c, \chi, \phi, \mu, \sigma^2) \times \phi y_H \frac{d\varrho}{d\chi} \tag{22}$$

where

$$\frac{dq(N^c, \chi, \phi, \mu, \sigma^2)}{d\chi} = -N^c f(\varrho)(\phi y_H \varrho - v)\frac{d\varrho}{d\chi} \tag{23}$$

A rise in fixed costs will require a greater pool of borrowers to cover them and this will have two opposite effects.

First, it will decrease the internal transfer price. Since more constrained households need to be pooled, the marginal borrower accepted will tend to be riskier. That is $\frac{d\varrho}{d\chi} < 0$, see proof in Appendix 8.6.2. This lower probability of repayment cutoff will yield a higher interest rate offered in the credit contract. Given default costs (ϕy_H), the vendor will optimally choose a lower internal transfer price and this decreases vendor financing gains.

Second, a rise in fixed costs will unambiguosly increase the number of credit borrowers, since $\left(\frac{dq(N^c,\chi,\phi,\mu,\sigma^2)}{d\chi}\right) > 0$. This will increase vendor financing gains.

Figures 7b and 7c illustrate the opposite effect on the internal transfer price and number of credit borrowers.

To pin down the net effect we need to know the size of marginal cost v relative to default costs ϕy_H. When v is higher than a given threshold v^*, a rise in fixed costs will reduce vendor financing gains. On the contrary, when v is lower than v^* a rise in fixed costs increases vendor financing gains. The marginal cost threshold is derived in Appendix 8.7.2 and defined as:

$$v^* = \left(\varrho - \frac{\int_{\varrho}^{1} f(\rho) d\rho}{f(\varrho)}\right) \phi y_H$$

In general, vendor financing gains after a marginal change in fixed costs decreases in the marginal cost. That is, given default costs ϕy_H, an increase in fixed costs will yield higher vendor financing incentives the lower is v.

6.4 A change in bankruptcy cost for consumer

Unlike previous comparative exercises, the partial derivative of vendor financing gains for case 1 and case 2 won't be equal, instead $\frac{dVF^{(1)}}{d\phi} < \frac{dVF^{(2)}}{d\phi}$.

We choose to illustrate case 1. The partial derivative of $VF^{(1)}$ with respect to default cost ϕ is defined as:

$$\frac{dVF^{(1)}}{d\phi} = \frac{dq(N^c,\chi,\phi,\mu,\sigma^2)}{d\phi} \times (\phi y_H \varrho - v) + q(N^c,\chi,\phi,\mu,\sigma^2)\phi y_H \frac{d\varrho}{d\phi} + q(N^c,\chi,\phi,\mu,\sigma^2)\varrho y_H \tag{24}$$

where

$$\frac{dq(N^c,\chi,\phi,\mu,\sigma^2)}{d\phi} = -N^c f(\varrho)\frac{d\varrho}{d\phi} \tag{25}$$

A rise in default costs will increase the maximum repayment value to be charged by the captive financial intermediary and this will have two opposite effects.

First, it will increase the maximum internal transfer price that vendors are able to charge and profit maximization would yield a higher z_1^c (figure 8b). This higher amount advanced in the first period by the captive financial intermediary, will require less constrained households to be pooled (figure 8a). Then the marginal borrower accepted will tend to be safer[10] and vendor financing gains increase.

Second, a rise in default costs will unambiguously decrease the number of credit borrowers, since $\left(\frac{dq(N^c,\chi,\phi,\mu,\sigma^2)}{d\phi}\right) < 0$. This will reduce vendor financing gains. Figure 8c illustrates the effect of a higher internal transfer price on the number of credit borrowers.

To pin down the net effect we need to know the size of marginal cost v relative to default costs ϕy_H. When v is higher than a given threshold v^{**}, a rise in default cost will increase vendor financing gains. On the contrary, when v is lower than v^{**} a rise in default costs reduces vendor financing gains. The marginal cost threshold is derived in Appendix 8.7.3 and defined as:

$$v^{**} = \left(\varrho - \frac{\varrho\phi y_H}{\chi} \times \frac{\left(\int_{\varrho}^{1} f(\rho)d\rho\right)^2}{f(\varrho)} - \frac{\int_{\varrho}^{1} f(\rho)d\rho}{f(\varrho)}\right)\phi y_H$$

In general, vendor financing gains after a marginal change in default costs increase in the marginal cost. That is, given default costs ϕy_H, an increase in default costs will yield higher vendor financing incentives the higher is v.

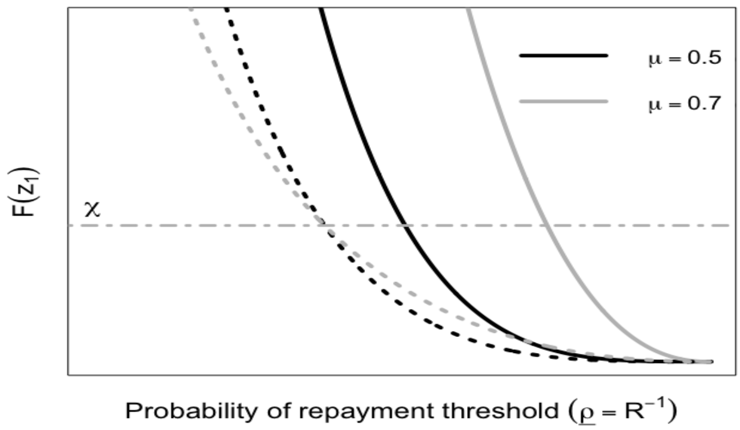
[10] This follows $\frac{d\varrho}{d\phi} > 0$, see proof in Appendix 8.6.3

6.5 A relative decrease in banks' fixed costs

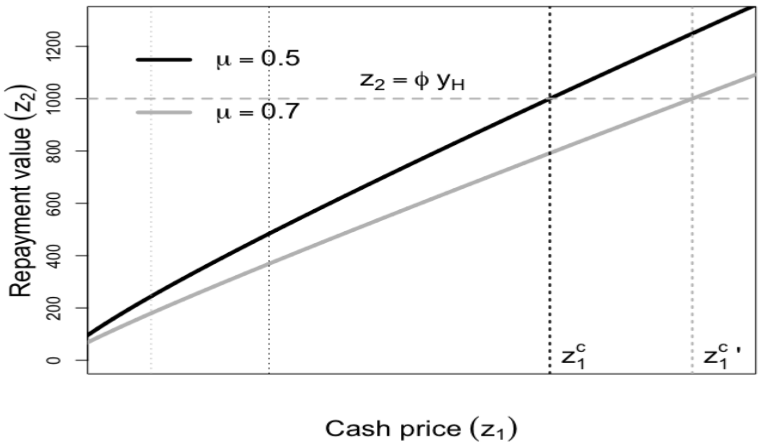
Work in progress.

Figure 6: An increase in the mean

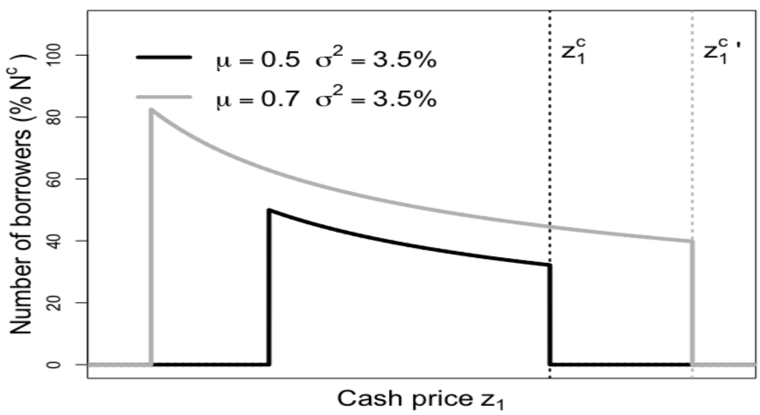
(a) Deriving $\varrho(z_1^c)$, given χ, ϕ and y_H



(b) Repayment value z_2 $(= z1/\varrho(z_1))$



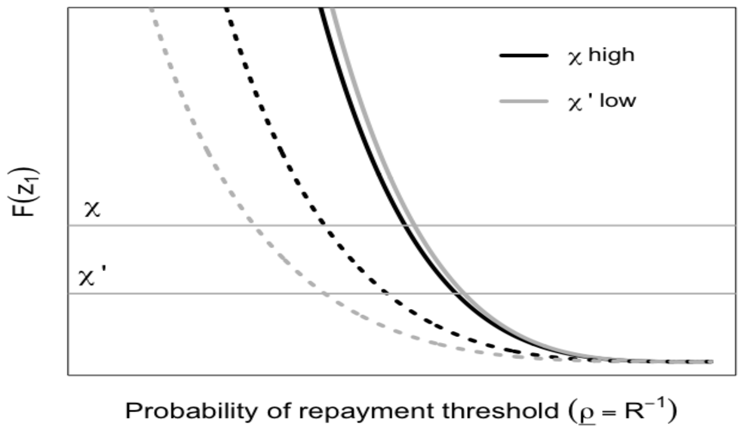
(c) Credit consumers (% of total constrained) in equilibrium



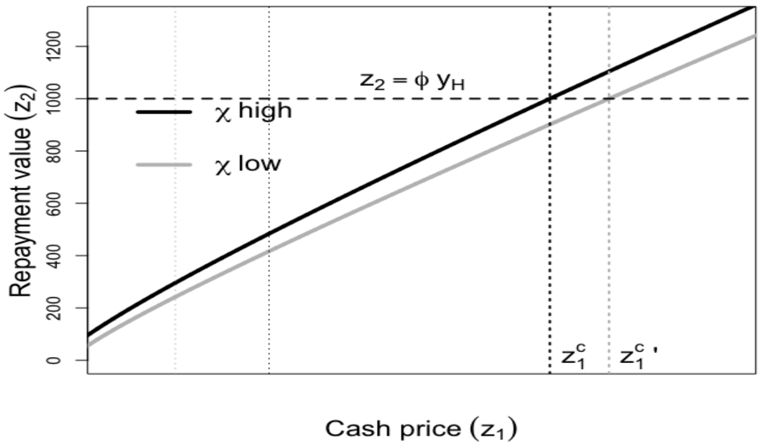
Continuous and dashed lines in (a) represent $F(z_1^c)$ and $F(z_1^{min})$ respectively. Value z_1^c satisfies $z_1^c = \phi y_H \varrho^c$ where ϱ^c is probability of repayment threshold at z_1^c. Value z_1^{min} is value of cash price at which its probability threshold corresponds to the inverse of ceiling rate (R^{max}). All subfigures assume $\chi = 40$, $\phi = 0.1$, $yh = 10000$ and $R^{max} = 2$

Figure 7: An decrease in financial sector's fixed cost

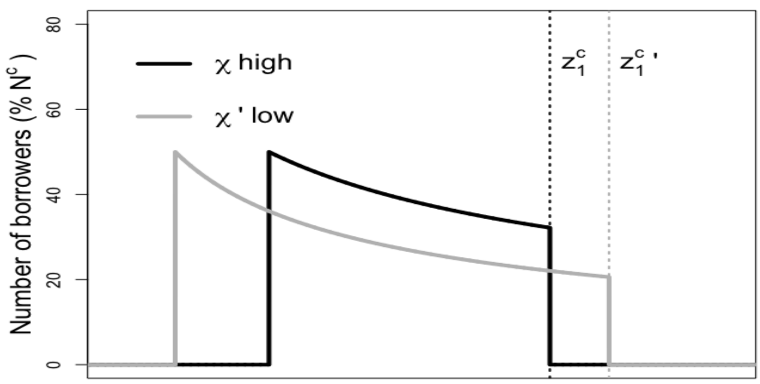
(a) Deriving $\varrho(z_1^c)$, given χ, ϕ and y_H



(b) Repayment value z_2 $(= z1/\varrho(z_1))$



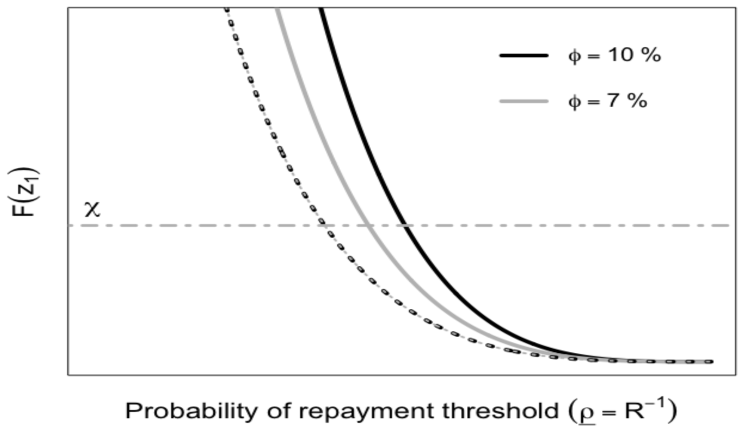
(c) Credit consumers (% of total constrained) in equilibrium



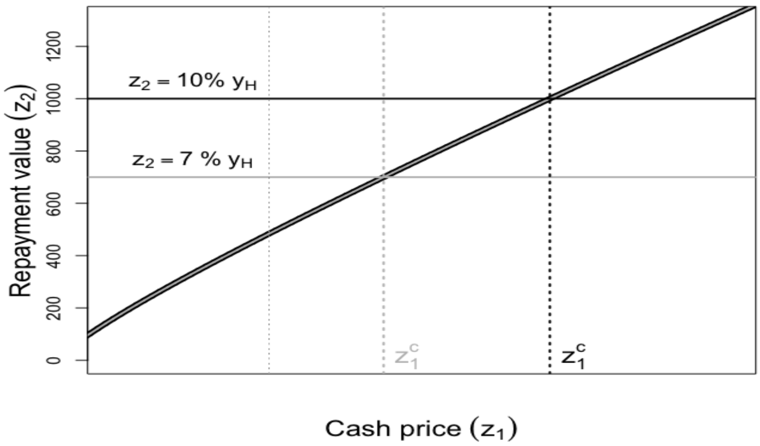
Continuous and dashed lines in (a) represent $F(z_1^c)$ and $F(z_1^{min})$ respectively. Value z_1^c satisfies $z_1^c = \phi y_H \varrho^c$ where ϱ^c is probability of repayment threshold at z_1^c. Value z_1^{min} is value of cash price at which its probability threshold corresponds to the inverse of ceiling rate (R^{max}). All subfigures assume $\phi = 0.1$, $yh = 10000$, $R^{max} = 2$, $E(\rho) = 0.5$ and $\sigma^2(\rho) = 3.5\%$

Figure 8: An rise in default costs

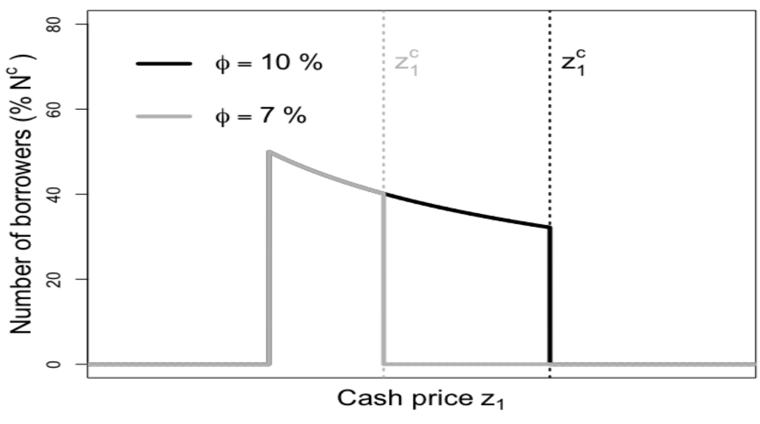
(a) Deriving $\underline{\rho}(z_1^c)$, given χ, ϕ and y_H



(b) Repayment value z_2 $(= z1/\underline{\rho}(z_1))$



(c) Credit consumers (% of total constrained) in equilibrium



Continuous and dashed lines in (a) represent $F(z_1^c)$ and $F(z_1^{min})$ respectively. Value z_1^c satisfies $z_1^c = \phi y_H \underline{\rho}^c$ where $\underline{\rho}^c$ is probability of repayment threshold at z_1^c. Value z_1^{min} is value of cash price at which its probability threshold corresponds to the inverse of ceiling rate (R^{max}). All subfigures assume $\chi = 40$, $yh = 10000$, $R^{max} = 2$, $E(\rho) = 0.5$ and $\sigma^2(\rho) = 3.5\%$

7 Testing some model implications and assumptions

7.1 Supporting bank framework

We differ from the stylized bank sector described in Brennan et al. (1988) for two reasons. First, banks in our model offer unsecured credit contracts. Second, banks in our model choose who to lend to and won't merely supply credit to everyone demanding it as Brennan et al. assumes.

If we use their setup, financial exclusion (measured by lack of use of bank credit services) could only be derived from lack of demand. In the most stylized version of our model, all constrained households demand credit but by allowing durable good stock heterogeneity we can easily extend it. Then, financial exclusion not only would derive from lack of demand but also from some supply barriers that impede individuals from accessing credit services. This is highly relevant as financial inclusion is a particularly important priority for developing and emerging countries of Latin America (García et al., 2013). Empirical evidence for the region suggests financial exclusion can't be attributed solely to barriers limiting credit demand or to those limiting supply, but rather is jointly determined by both (Rojas-Suarez and Amado, 2014).

Fixed costs is the key mechanism that leads banks in our setup to choose who receives credit and through which supply barriers arise. Unlike in Brennan et al. where lenders never know borrower's risk type, in our model, banks can have some information (in this case, perfect) after paying this fixed cost.

There is supporting evidence that fixed costs for banks in the region are significantly high. A common indicator of banks' operational inefficiency is the ratio of overhead (administrative) costs to total assets. High ratios tend to increase the fixed costs of extending loans. Rojas-Suarez and Amado (2014) find that the median value for Latin America is over 50 percent higher than the median value for countries with similar real income per capita. This evidence supports our setup over the simple framework of Brennan with no fixed costs.

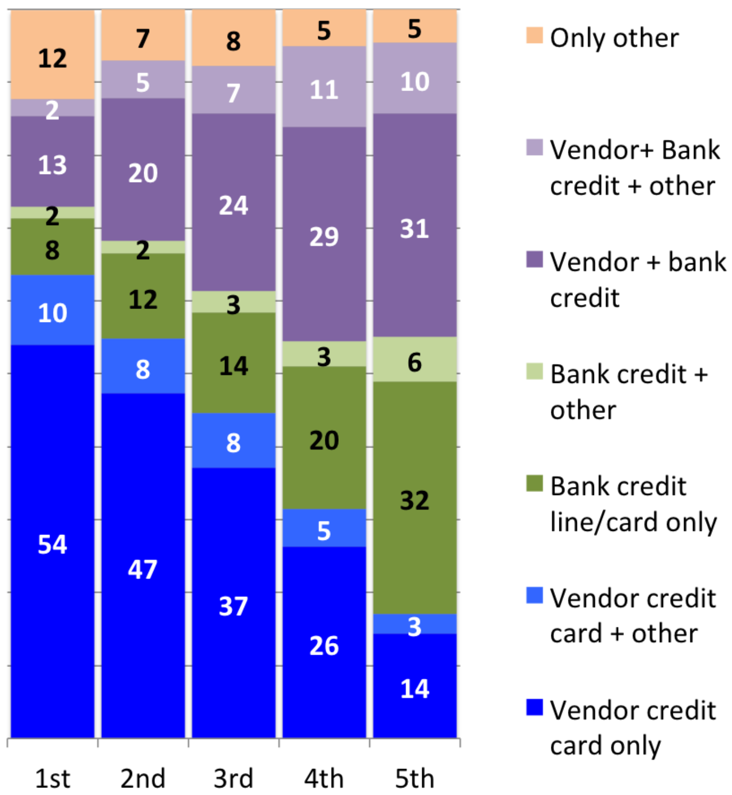
Finally, our model allow us to derive implications of reducing fixed costs on the percentage of constrained households using credit services. This is an interesting comparative statics exercise since high operational costs is one of many causes of financial exclusion in Latin America.

7.2 Vendor supplies credit to financially constrained households only

An assumption by the model is that vendors wish to offer credit contracts only to households in need of credit to purchase their goods, not those that can pay with cash. In addition, their optimization problem leads them to offer better credit contract terms than banks. Put together, we expect to see a higher percentage of households holding vendor credit at the lower quintiles of income distribution.

To check this we use the Household financial survey 2007 conducted by the Central Bank of Chile and measure the percentage of total households holding vendor credit by income quintile. Figure 9 shows that commercial stores are the main consumption credit provider for the lower income quintiles in Chile. On the contrary, there is greater tendency to hold bank credit in the form of credit lines or credit cards as income increases. Interestingly, as income increases the tendency to hold both types of lending also increases.

Figure 9: Source of consumption credit by income quintile (% of reporting Households in 2007)



Source: Household financial survey 2007. Central Bank of Chile
Note: Other credit is the sum of educational credit, auto loans and other credit provided by the government for social purposes.

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8 Appendix

8.1 Maximum durable good stock of constrained households

Definition 8.1. *Durable good stock d_0^{max} is the maximum level at which households -regardless their probability of repayment- are indifferent between accepting credit offer to purchase durable good or rejecting it and therefore not buying it.*

$$\frac{(1-\phi)^{\frac{\beta\ \gamma}{(1-\gamma)(1+\beta)}}}{1-(1-\phi)^{\frac{\beta\ \gamma}{(1-\gamma)(1+\beta)}}} = (1-\delta)d_0^{max}$$

We will proceed to derive this. Remember period utility take Cobb-Douglas functional form:

$$U(c_t, d_t)) = \frac{\left(c_t^{\gamma} d_t^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}$$

Consider its log transformation:

$$u(c_t, d_t)) = log(U(c_t, d_t)) = \psi_c log(c_t) + \psi_d log(d_t) - log(\psi_c + \psi_d)$$

where $\psi_c = (1-\sigma)\gamma$, $\psi_d = (1-\sigma)(1-\gamma)$ and $\psi_c + \psi_d = 1-\sigma$.

The value of autarky is

$$v^{nb}(d_0, \rho) = u(y_1, (1-\delta)d_0) + \beta\rho\, u(y_H, (1-\delta)^2 d_0) + \beta(1-\rho)\, u(y_L, (1-\delta)^2 d_0)$$

The value of accepting credit offer to purchase durable good is

$$\begin{aligned} v^{b}(d_0, \rho, z_2) =& u(y_1, 1 + (1-\delta)d_0) + \beta\rho\, u(y_H - z_2, (1-\delta) + (1-\delta)^2 d_0) + \dots \\ &+ \beta(1-\rho)\, u((1-\phi)y_L, (1-\delta) + (1-\delta)^2 d_0) \end{aligned}$$

Given durable good stock and probability of repayment, if $v^b(d_0, \rho, z_2) \geq v^{nb}(d_0, \rho)$, then household accepts credit offer and purchases one more unit of durable good.

Simplifying and rearranging this expression yields

$$\begin{aligned} &\psi_d(1+\beta)\, log\left(\frac{1+(1-\delta)d_0}{(1-\delta)d_0}\right) + \beta\psi_c\, log(1-\phi) \\ \geq& \\ &\beta\rho\, \psi_c\, log\left(\frac{y_H}{y_H - z_2}(1-\phi)\right) \end{aligned}$$

Note left hand side of equation above is decreasing in durable good stock. Then there will be a maximum d_0 such that for all values below it, $v^b(d_0, \rho, z_2) \geq v^{nb}(d_0, \rho)$. The value d_0^{max} solves:

$$\psi_d(1+\beta)\, log\left(\frac{1+(1-\delta)d_0^{max}}{(1-\delta)d_0^{max}}\right) + \beta\psi_c\, log(1-\phi) = \beta\, \psi_c\, log\left(\frac{y_H}{y_H - z_2}(1-\phi)\right)$$

Substituting $y_H(1-\phi) = y_H - z_2$ and solving for d_0^{max} yields the following:

$$\frac{(1-\phi)^{\frac{\beta\,\gamma}{(1-\gamma)(1+\beta)}}}{1-(1-\phi)^{\frac{\beta\,\gamma}{(1-\gamma)(1+\beta)}}} = (1-\delta)d_0^{max}$$

If $d_0 > d_0^{max}$, then value of rejecting credit offer is higher than value of accepting it, regardless the probability of repayment.

8.2 Repayment value is an increasing function of z_1

The repayment value z_2 can be expressed as $z_1 \times R$, where $R = 1/\underline{\rho}$. Note $\underline{\rho}$ is the probability of repayment threshold derived from the bank problem.

$$\frac{dz_2}{dz_1} = \frac{1}{\underline{\rho}} - \frac{z_1}{\underline{\rho}^2}\frac{d\underline{\rho}}{dz_1} = \frac{1}{\underline{\rho}}\left(1 - \frac{z_1}{\underline{\rho}}\frac{d\underline{\rho}}{dz_1}\right)$$

To get $\frac{d\underline{\rho}(z_1)}{dz_1}$ we apply the implicit function theorem on Bank Profits:

$$\Pi^B = \int_{\underline{\rho}(z_1)}^{\bar{a}} \left(\frac{\rho}{\underline{\rho}(z_1)} - 1\right) z_1 \times f(\rho)d\rho - \chi = 0$$

$$\Pi^B = -\frac{1}{\underline{\rho}(z_1)}\int_{\bar{a}}^{\underline{\rho}(z_1)} \rho z_1 \times f(\rho)d\rho + \int_{\bar{a}}^{\underline{\rho}(z_1)} z_1 \times f(\rho)d\rho - \chi$$

$$\frac{d\Pi^B}{dz_1} = \int_{\underline{\rho}}^{\bar{a}} \left(\frac{\rho}{\underline{\rho}} - 1\right) \times f(\rho)d\rho$$

$$\frac{d\Pi^B}{d\underline{\rho}} = \frac{1}{\underline{\rho}^2}\int_{\bar{a}}^{\underline{\rho}} \rho z_1 \times f(\rho)d\rho$$

$$\frac{d\underline{\rho}}{dz_1} = -\frac{d\Pi^B/dz_1}{d\Pi^B/d\underline{\rho}}$$

$$\frac{d\underline{\rho}}{dz_1} = \frac{\underline{\rho}^2\int_{\underline{\rho}}^{\bar{a}}\left(\frac{\rho}{\underline{\rho}} - 1\right) \times f(\rho)d\rho}{z_1\int_{\underline{\rho}}^{\bar{a}} \rho \times f(\rho)d\rho} = \frac{\underline{\rho}\int_{\underline{\rho}}^{\bar{a}} (\rho - \underline{\rho}) \times f(\rho)d\rho}{z_1\int_{\underline{\rho}}^{\bar{a}} \rho \times f(\rho)d\rho} > 0$$

Substituting in dz_2/dz_1 yields

$$\frac{dz_2}{dz_1} = \frac{1}{\underline{\rho}} - \frac{z_1}{\underline{\rho}^2}\frac{d\underline{\rho}}{dz_1} = \frac{1}{\underline{\rho}}\left(1 - \frac{\int_{\underline{\rho}}^{\bar{a}} (\rho - \underline{\rho}) \times f(\rho) d\rho}{\int_{\underline{\rho}}^{\bar{a}} \rho \times f(\rho) d\rho}\right) > 0$$

8.3 Constraint: Cash customer doesn't buy on credit

We will prove the following: "*If $z_1' \frac{R^V}{R_r^B} > z_1$, cash customer won't buy on credit provided by vendor*"

Assume $z_1' \frac{R^V}{R_r^B} > z_1$ and cash customer finances the purchase of one unit of durable good with credit provided by vendor. Let c_1^* and c_2^* be the optimal first and second period non-durable consumption. Then the first period budget constraint is

$$c_1^* + z_1' = y_1 + b_1 + b_1^V$$

where b_1 is total consumption credit provided by banks and b_1^V is consumption credit by vendor.

The second period budget constraint is

$$c_2^* = y_2 - b_1 R_r^B - b_1^V R^V$$

Substituting b_1 in first period budget constraint yields the lifetime budget constraint

$$c_1^* + z_1' + \frac{c_2^*}{R_r^B} = y_1 + \frac{y_2}{R_r^B} + b_1^V\left(\frac{R_r^B - R^V}{R_r^B}\right)$$

Note that $b_1^V = z_1'$ since credit provided by vendors covers exactly the subsidized price. Then

$$c_1^* + \frac{c_2^*}{R_r^B} = y_1 + \frac{y_2}{R_r^B} - z_1'\left(\frac{R^V}{R_r^B}\right)$$

Recall that lifetime budget constraint of cash customer not financing purchase of durable good with vendor credit is

$$c_1 + \frac{c_2}{R_r^B} = y_1 + \frac{y_2}{R_1^B} - z_1$$

Given assumption $z_1' \frac{R^V}{R_r^B} > z_1$,

$$c_1 + \frac{c_2}{R_r^B} > c_1^* + \frac{c_2^*}{R_r^B}$$

a contradiction to the statement that cash customer will finance the purchase of one unit of durable good with credit provided by vendor.

8.4 Characterizing profits from selling to credit customers

Profits from selling to credit customers is defined as

$$\Pi(z_1) = q(z_1, \chi)(z_1 - v)$$

First derivative with respect to z_1

$$\begin{aligned}
\frac{d\Pi(z_1)}{dz_1} &= q(z_1, \chi) + (z_1 - v)\frac{dq(z_1, \chi)}{dz_1} \\
&= N^c \times (1 - G(\underline{\rho}(z_1))) - (z_1 - v)N^c \frac{dG(\underline{\rho}(z_1)}{d\underline{\rho}} \frac{d\underline{\rho}}{dz_1} \\
&= N^c \times (1 - G(\underline{\rho}(z_1))) - (z_1 - v)N^c f(\underline{\rho}(z_1)) \frac{d\underline{\rho}}{dz_1}
\end{aligned}$$

To get $\frac{d\underline{\rho}(z_1)}{dz_1}$ we apply the implicit function theorem on Bank Profits and get

$$\frac{d\underline{\rho}}{dz_1} = \frac{\underline{\rho}^2 \int_{\underline{\rho}}^{\bar{a}} \left(\frac{\rho}{\underline{\rho}} - 1\right) \times f(\rho)d\rho}{z_1 \int_{\underline{\rho}}^{\bar{a}} \rho \times f(\rho)d\rho} > 0$$

Next we prove that

$$\frac{d\Pi(z_1)}{dz_1} > 0$$

$$\frac{d\Pi(z_1)}{dz_1} = N^c \times (1 - G(\underline{\rho}(z_1))) - (z_1 - v)N^c f(\underline{\rho}(z_1)) \left(\frac{\underline{\rho}^2 \int_{\underline{\rho}}^{\bar{a}} \left(\frac{\rho}{\underline{\rho}} - 1\right) \times f(\rho)d\rho}{z_1 \int_{\underline{\rho}}^{\bar{a}} \rho \times f(\rho)d\rho}\right)$$

$$\frac{d\Pi(z_1)}{dz_1} = N^c \left(\int_{\underline{\rho}}^{\bar{a}} f(\rho)d\rho\right) - (z_1 - v)N^c f(\underline{\rho}(z_1)) \left(\frac{\underline{\rho}^2 \int_{\underline{\rho}}^{\bar{a}} \left(\frac{\rho}{\underline{\rho}} - 1\right) \times f(\rho)d\rho}{z_1 \int_{\underline{\rho}}^{\bar{a}} \rho \times f(\rho)d\rho}\right)$$

$$\frac{d\Pi(z_1)}{dz_1} = N^c \left(\int_{\underline{\rho}}^{\bar{a}} f(\rho)d\rho - (z_1 - v) f(\underline{\rho}(z_1)) \left(\frac{\underline{\rho}^2 \int_{\underline{\rho}}^{\bar{a}} \left(\frac{\rho}{\underline{\rho}} - 1\right) \times f(\rho)d\rho}{z_1 \int_{\underline{\rho}}^{\bar{a}} \rho \times f(\rho)d\rho}\right)\right)$$

Note both relations below

$$\int_{\underline{\rho}}^{\bar{a}} f(\rho)d\rho > \int_{\underline{\rho}}^{\bar{a}} \rho f(\rho)d\rho$$

$$\frac{z_1 - v}{z_1} f(\underline{\rho}(z_1))\underline{\rho} > \frac{z_1 - v}{z_1} f(\underline{\rho}(z_1)) \left(\frac{\underline{\rho} \int_{\underline{\rho}}^{\bar{a}} (\rho - \underline{\rho}) \times f(\rho)d\rho}{\int_{\underline{\rho}}^{\bar{a}} \rho \times f(\rho)d\rho}\right)$$

Since...

$$\frac{\int_{\underline{\rho}}^{\bar{a}} \rho f(\rho)d\rho}{f(\underline{\rho})\underline{\rho}} > 1 \Rightarrow \frac{\int_{\underline{\rho}}^{\bar{a}} \rho f(\rho)d\rho}{f(\underline{\rho})\underline{\rho}} > \frac{z_1 - v}{z_1}$$

We can prove that

$$\int_{\underline{\rho}}^{\bar{a}} f(\rho)d\rho > \frac{z_1 - v}{z_1} f(\underline{\rho}(z_1)) \left(\frac{\underline{\rho} \int_{\underline{\rho}}^{\bar{a}} (\rho - \underline{\rho}) \times f(\rho)d\rho}{\int_{\underline{\rho}}^{\bar{a}} \rho \times f(\rho)d\rho} \right)$$

This implies $\frac{d\Pi(z_1)}{dz_1} > 0$

8.5 Restriction on z_1 so that cash customer purchases

8.5.1 Deriving optimal non-durable consumption for the cash customer

In their two period optimization problem, unconstrained households choose non-durable consumption for their two periods (c_1, c_2) and whether they purchase one unit of durable good in the first period or not. That is, they maximize utility:

$$\begin{aligned} \max_{\{c_1, c_2, \textit{Purchase or No Purchase}\}} \quad & u(c_1, d_1) + \beta u(c_2, d_2) \\ \textit{subject to:} \quad & c_1 + z_1 x_1 + \frac{c_2}{R_1^B} = y_1 + \frac{y_2}{R_1^B} \\ & d_1 = x_1 + (1-\delta)d_0 \end{aligned} \tag{26}$$

where z_1 is the relative price of durable goods, x_1 is units of durable goods purchased. Remember we assume each household can only but one unit of durable good.

The first order condition for c_1 yields:

$$\begin{aligned} u_1(c_1^*, d_1) =& \beta u_1(c_2^*, d_2) R_1^B \\ \textit{with: } c_2^* =& R_1^B (y_1 - c_1^* - z_1 x_1) + y_2 \end{aligned}$$

where u_1 be the derivative of the transformed period utility function.

Let c_1^{p*} be optimal non-durable consumption if a household purchases one unit of durable good. Then, c_1^{p*} solves

$$\begin{aligned} u_1(c_1^{p*}, 1 + (1-\delta)d_0) =& \beta u_1(c_2^{p*}, (1-\delta) + (1-\delta)^2 d_0) R_1^B \\ \textit{with: } c_2^{p*} =& R_1^B (y_1 - c_1^{p*} - z_1) + y_2 \end{aligned} \tag{27}$$

Let c_1^{np*} be optimal non-durable consumption if household doesn't purchase any durable good. Then, c_1^{np} solves

$$\begin{aligned} u_1(c_1^{np*}, (1-\delta)d_0) =& \beta u_1(c_2^{np*}, (1-\delta)^2 d_0) R_1^B \\ \textit{with: } c_2^{np*} =& R_1^B (y_1 - c_1^{np*}) + y_2 \end{aligned} \tag{28}$$

Assuming period utility takes the Cobb-Douglas functional form (in logs):

$$u(c_t, d_t)) = \psi_c log(c_t) + \psi_d log(d_t) - log(\psi_c + \psi_d)$$

Then

$$u_1(c_t, d_t)) = \frac{\psi_c}{c_t} \tag{29}$$

Substitute (11) in (9) and the F.O.C solving for c_1^{p*} is:

$$\begin{aligned}\frac{\psi_c}{c_1^{p*}} =& \beta \frac{\psi_c}{c_2^{p*}} R_1^B \\ \textit{with: } c_2^{p*} =& R_1^B(y_1 - c_1^{p*} - z_1) + y_2\end{aligned} \tag{30}$$

Eliminating common terms and rearranging:

$$c_1^{p*} = \frac{1}{(1+\beta)} \left[y_1 - z_1 + \frac{y_2}{R_1^B} \right] \tag{31}$$

Substitute (11) in (10) and the F.O.C solving for c_1^{np*} is:

$$\begin{aligned}\frac{\psi_c}{c_1^{np*}} =& \beta \frac{\psi_c}{c_2^{np*}} R_1^B \\ \textit{with: } c_2^{np*} =& R_1^B(y_1 - c_1^{np*}) + y_2\end{aligned} \tag{32}$$

Eliminating common terms and rearranging:

$$c_1^{np*} = \frac{1}{(1+\beta)} \left[y_1 + \frac{y_2}{R_1^B} \right] \tag{33}$$

8.5.2 Deriving restriction on z_1

We consider the log transformation of Cobb-Douglas utility function

$$u(c_t, d_t)) = log(U(c_t, d_t)) = \psi_c log(c_t) + \psi_d log(d_t) - log(\psi_c + \psi_d) \tag{34}$$

where $\psi_c = (1-\sigma)\gamma$, $\psi_d = (1-\sigma)(1-\gamma)$ and $\psi_c + \psi_d = 1 - \sigma$.

Let v_r^b be the value of purchasing the good:

$$\begin{aligned} v_r^b =& u(c_1^{p*}, 1 + (1-\delta)d_0) + \beta u(R_1^B(y_1 - c_1^{p*} - z_1) + y_2, (1-\delta) + (1-\delta)^2 d_0) \\ =& \psi_c \, log(c_1^{p*}) + \psi_d \, log(1 + (1-\delta)d_0) - log(\psi_c + \psi_d) + \ldots \\ & + \beta\psi_c \, log(R_1^B(y_1 - c_1^{p*} - z_1) + y_2) + \beta\psi_d \, log((1-\delta) + (1-\delta)^2 d_0) - \beta \, log(\psi_c + \psi_d) \end{aligned} \tag{35}$$

where $c_1^{p*} = \frac{1}{(1+\beta)} \left[y_1 - z_1 + \frac{y_2}{R_1^B} \right]$

Let v_r^{nb} be the value of not purchasing the good:

$$\begin{aligned} v_r^{nb} =& u(c_1^{np*}, (1-\delta)d_0) + \beta u(R_1^B(y_1 - c_1^{np*}) + y_2, (1-\delta)^2 d_0)) \\ =& \psi_c\, log(c_1^{np*}) + \psi_d\, log((1-\delta)d_0) - log(\psi_c + \psi_d) + \ldots \\ & + \beta\psi_c\, log(R_1^B(y_1 - c_1^{np*}) + y_2) + \beta\psi_d\, log((1-\delta)^2 d_0) - \beta\, log(\psi_c + \psi_d) \end{aligned} \tag{36}$$

where $c_1^{np*} = c_1^{np*} = \frac{1}{(1+\beta)}\left[y_1 + \frac{y_2}{R_1^B}\right]$

An unconstrained household will choose to purchase one unit of durable good as long as:

$$v_r^b \geq v_r^{nb}$$

Substituting in c_1^{p*} and c_1^{np*},rearranging and using log properties yields

$$\begin{aligned} & \psi_c\, log\left(1 - \frac{z_1}{y_1 + \frac{y_2}{R_1^B}}\right) + \ldots \\ & + \beta\psi_c\, log\left(1 - \frac{\beta z_1 R_1^B/(1+\beta)}{R_1^B(y_1 - \frac{1}{(1+\beta)}\left[y_1 + \frac{y_2}{R_1^B}\right]) + y_2}\right) \\ \geq & \\ & \psi_d\, log\left(\frac{(1-\delta)d_0}{1 + (1-\delta)d_0}\right) + \ldots \\ & + \beta\psi_d\, log\left(\frac{(1-\delta)^2 d_0}{(1-\delta) + (1-\delta)^2 d_0})\right) \end{aligned}$$

Simplifying

$$log\left(1 - \frac{z_1}{y_1 + \frac{y_2}{R_1^B}}\right) \geq -\frac{\psi_d}{\psi_c}\, log\left(\frac{1}{(1-\delta)d_0} + 1\right)$$

Exponentiation of both sides

$$\left(y_1 + \frac{y_2}{R_1^B}\right)\left(1 - \left(\frac{1}{(1-\delta)d_0} + 1\right)^{-\frac{\psi_d}{\psi_c}}\right) \geq z_1$$

Let $y_1 = y_2 = \bar{y}$, and since $\frac{\psi_d}{\psi_c} = \frac{(1-\gamma)}{\gamma}$ then:

$$\frac{(1 + R_1^B)}{R_1^B}\bar{y}\;\; \Omega(d_0, \delta, \gamma) \geq z_1 \tag{37}$$

where

$$\Omega(d_0, \delta, \gamma) = 1 - \left(\frac{1}{(1-\delta)d_0} + 1\right)^{-\frac{\psi_d}{\psi_c}}$$

Note

$$\frac{d(\Omega(d_0,\delta,\gamma))}{d(d_0)} = \frac{1-\gamma}{\gamma}\left(\frac{1}{(1-\delta)d_0}\right)^{-\frac{1}{\gamma}} \frac{(-1)}{(1-\delta)d_0^2} < 0$$

That is, the greater d_0 (or the lower $\bar{y}$), the lower is the upper bound of price z_1 such that cash customer accepts to purchase the durable good.

8.6 Analytical derivatives

8.6.1 Derivative of probability of repayment threshold with respect to $E(\rho)$

To get $\frac{d\underline{\rho}}{dE(\rho)}$, we use the implicit function theorem on bank's optimality equation

$$F = \int_{\underline{\rho}}^{1} (\rho - \underline{\rho}) \times f(\rho) d\rho - \frac{\chi}{\phi y_H} = 0 \tag{38}$$

$$\frac{d\underline{\rho}}{dE(\rho)} = -\frac{dF/dE(\rho)}{dF/d\underline{\rho}} = \frac{\int_{\underline{\rho}}^{1} (\rho - \underline{\rho}) \times \frac{df(\rho)}{dE(\rho)} d\rho}{\int_{\underline{\rho}}^{1} f(\rho) d\rho} > 0 \tag{39}$$

where $\rho \sim Beta(\alpha, \beta)$, $\frac{df(\rho)}{dE(\rho)}$ is the derivative of the density with respect to the mean of the distribution.

8.6.2 Derivative of probability of repayment threshold with respect to χ

To get $\frac{d\underline{\rho}}{d\chi}$ we use the implicit function theorem on equation 38.

$$\frac{d\underline{\rho}}{d\chi} = -\frac{dF/d\chi}{dF/d\underline{\rho}} = \frac{-(\phi y_H)^{-1}}{\int_{\underline{\rho}}^{1} f(\rho) d\rho} < 0 \tag{40}$$

8.6.3 Derivative of probability of repayment threshold with respect to ϕ

To get $\frac{d\underline{\rho}}{d\phi}$ we use the implicit function theorem on equation 38.

$$\frac{d\underline{\rho}}{d\phi} = -\frac{dF/d\phi}{dF/d\underline{\rho}} = \frac{\chi/(y_H \phi^2)}{\int_{\underline{\rho}}^{1} f(\rho) d\rho} > 0 \tag{41}$$

8.7 Deriving marginal cost threshold for comparative statics

8.7.1 Rise in the mean section

Let

$$\frac{dVF}{dE(\rho)} = N^c \left((\phi y_H \underline{\rho} - v) \int_{\underline{\rho}}^{1} \frac{df(\rho)}{d\mu} d\rho + \left(\phi y_H \int_{\underline{\rho}}^{1} f(\rho) d\rho - f(\underline{\rho})(\phi y_H \underline{\rho} - v) \right) \times \frac{\int_{\underline{\rho}}^{1} (\rho - \underline{\rho}) \times \frac{df(\rho)}{d\mu} d\rho}{\int_{\underline{\rho}}^{1} f(\rho) d\rho} \right) \tag{42}$$

Equivalently,

$$\frac{dVF}{dE(\rho)} = \frac{N^c}{\int_{\varrho}^1 f(\rho)d\rho}\left((\phi y_H \varrho - v)\int_{\varrho}^1 \frac{df(\rho)}{d\mu}d\rho \int_{\varrho}^1 f(\rho)d\rho + \left(\phi y_H \int_{\varrho}^1 f(\rho)d\rho - f(\varrho)(\phi y_H \varrho - v)\right) \times \int_{\varrho}^1 (\rho - \varrho) \times \frac{df(\rho)}{d\mu}d\rho\right)$$

For $\frac{dVF}{dE(\rho)} > 0$ we need

$$\begin{aligned}\int_{\varrho}^1 f(\rho)d\rho\left(\frac{\phi y_H \varrho - v}{\phi y_H}\int_{\varrho}^1 \frac{df(\rho)}{d\mu}d\rho + \int_{\varrho}^1 (\rho - \varrho) \times \frac{df(\rho)}{d\mu}d\rho\right) \\ > \frac{\phi y_H \varrho - v}{\phi y_H} f(\varrho)\int_{\varrho}^1 (\rho - \varrho) \times \frac{df(\rho)}{d\mu}d\rho\end{aligned}$$

$$\frac{\left(\int_{\varrho}^1 f(\rho)d\rho\right) \times \int_{\varrho}^1 (\rho - \varrho) \times \frac{df(\rho)}{d\mu}d\rho}{\left(\int_{\varrho}^1 f(\rho)d\rho \int_{\varrho}^1 \frac{df(\rho)}{d\mu}d\rho + f(\varrho)\int_{\varrho}^1 (\rho - \varrho) \times \frac{df(\rho)}{d\mu}d\rho\right)}\phi y_H < -\varrho\phi y_H + v \tag{43}$$

$$\varrho + \frac{\left(\int_{\varrho}^1 f(\rho)d\rho\right) \times \int_{\varrho}^1 (\rho - \varrho) \times \frac{df(\rho)}{d\mu}d\rho}{\left(\int_{\varrho}^1 f(\rho)d\rho \int_{\varrho}^1 \frac{df(\rho)}{d\mu}d\rho + f(\varrho)\int_{\varrho}^1 (\rho - \varrho) \times \frac{df(\rho)}{d\mu}d\rho\right)} < \frac{v}{\phi y_H} \tag{44}$$

8.7.2 Fixed cost section

Let

$$\begin{aligned}\frac{dVF}{d\chi} &= -N^c f(\varrho)\frac{d\varrho}{d\chi} \times (\phi y_H \varrho - v) + q(N^c, \chi, \phi, \mu, \sigma^2) \times \phi y_H \frac{d\varrho}{d\chi} \\ &= N^c\left(-f(\varrho)(\phi y_H \varrho - v) + \int_{\varrho}^1 f(\rho)d\rho \times \phi y_H\right)\frac{d\varrho}{d\chi}\end{aligned} \tag{45}$$

Since $\frac{d\varrho}{d\chi} < 0$, $\frac{dVF}{d\chi} < 0$ if,

$$\frac{\int_{\varrho}^1 f(\rho)d\rho}{f(\varrho)} > \varrho - \frac{v}{\phi y_H}$$

Equivalently,

$$\left(\varrho - \frac{\int_{\varrho}^1 f(\rho)d\rho}{f(\varrho)}\right)\phi y_H < v$$

8.7.3 Default cost section

Let

$$\frac{dVF}{d\phi} = \frac{dq(N^c, \chi, \phi, \mu, \sigma^2)}{d\phi} \times (\phi y_H \varrho - v) + q(N^c, \chi, \phi, \mu, \sigma^2)\phi y_H \frac{d\varrho}{d\phi} + q(N^c, \chi, \phi, \mu, \sigma^2)\varrho y_H \tag{46}$$

Substitute in terms and yields

$$
\begin{aligned}
\frac{dVF}{d\phi} &= N^c \left(-f(\underline{\rho})(\phi y_H \underline{\rho} - v) + \int_{\underline{\rho}}^{1} f(\rho) d\rho \times \phi y_H \right) \frac{d\underline{\rho}}{d\phi} + N^c \underline{\rho} y_H \times \int_{\underline{\rho}}^{1} f(\rho) d\rho \\
&= N^c \left(-f(\underline{\rho})(\phi y_H \underline{\rho} - v) + \int_{\underline{\rho}}^{1} f(\rho) d\rho \times \phi y_H \right) \frac{\chi/(y_H \phi^2)}{\int_{\underline{\rho}}^{1} f(\rho) d\rho} + N^c \underline{\rho} y_H \times \int_{\underline{\rho}}^{1} f(\rho) d\rho \\
&= N^c \left(-f(\underline{\rho})(\phi y_H \underline{\rho} - v) + \int_{\underline{\rho}}^{1} f(\rho) d\rho \times \phi y_H \right) \frac{\chi}{(y_H \phi^2) \int_{\underline{\rho}}^{1} f(\rho) d\rho} + N^c \underline{\rho} y_H \times \int_{\underline{\rho}}^{1} f(\rho) d\rho \\
&= \frac{N^c}{(y_H \phi^2) \int_{\underline{\rho}}^{1} f(\rho) d\rho} \left(\left(-f(\underline{\rho})(\phi y_H \underline{\rho} - v) + \int_{\underline{\rho}}^{1} f(\rho) d\rho \times \phi y_H \right) \chi + \underline{\rho} (\phi y_H)^2 \times \left(\int_{\underline{\rho}}^{1} f(\rho) d\rho \right)^2 \right)
\end{aligned} \tag{47}
$$

$\frac{dVF}{d\phi} > 0$ if,

$$
\frac{\underline{\rho} \phi y_H}{\chi} \times \frac{\left(\int_{\underline{\rho}}^{1} f(\rho) d\rho \right)^2}{f(\underline{\rho})} + \frac{\int_{\underline{\rho}}^{1} f(\rho) d\rho}{f(\underline{\rho})} > \underline{\rho} - \frac{v}{\phi y_H}
$$

Equivalently,

$$
\left(\underline{\rho} - \frac{\underline{\rho} \phi y_H}{\chi} \times \frac{\left(\int_{\underline{\rho}}^{1} f(\rho) d\rho \right)^2}{f(\underline{\rho})} - \frac{\int_{\underline{\rho}}^{1} f(\rho) d\rho}{f(\underline{\rho})} \right) \phi y_H < v
$$

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