

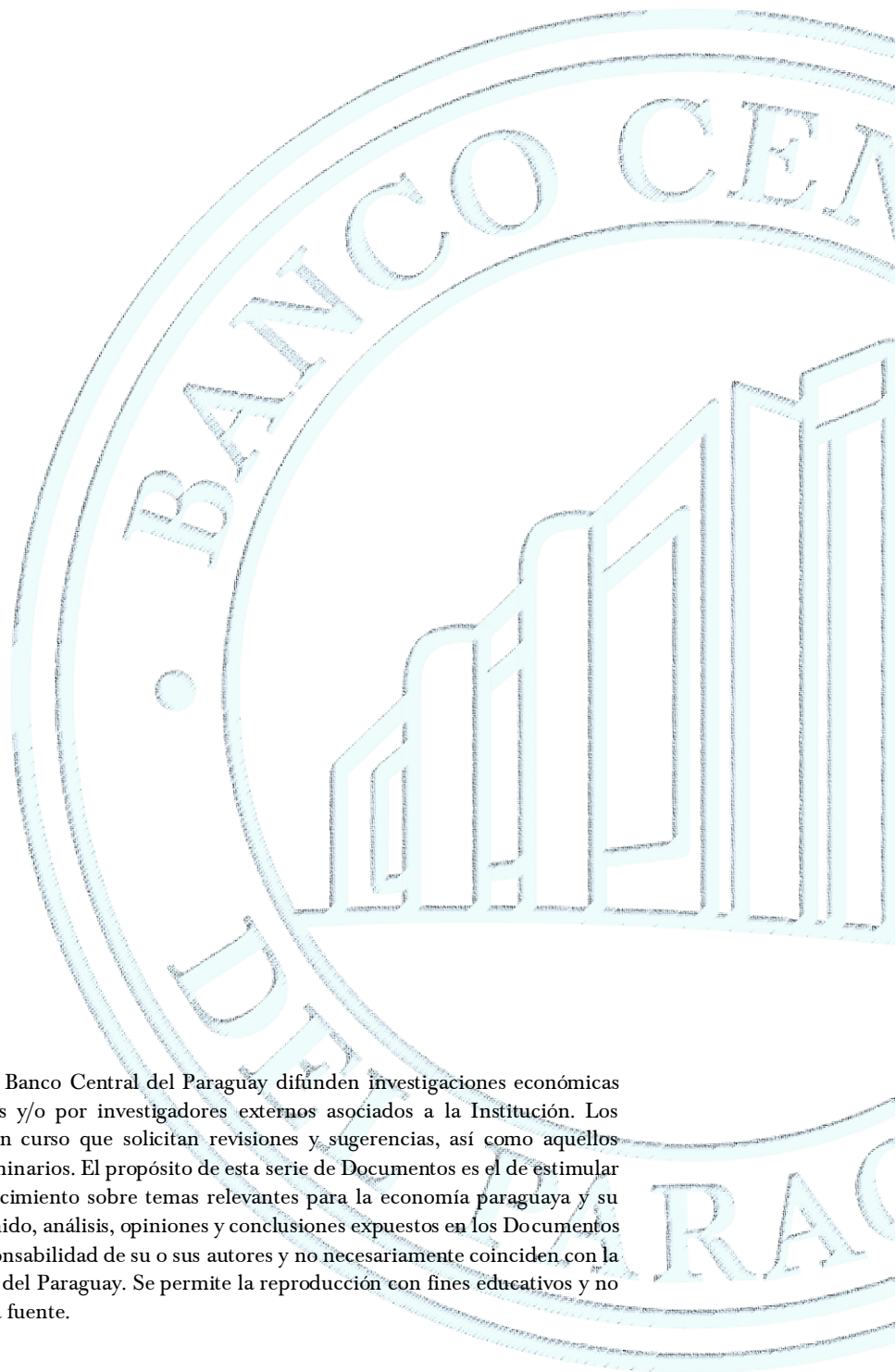
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# MONETARY POLICY IN A SMALL OPEN ECONOMY: THE CASE OF PARAGUAY

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## 1. Introduction

The main motivation of this research work is that the Central Bank of Paraguay has announced a change in the current monetary policy objective based on the control of monetary aggregates, to an inflation targeting approach. The monetary authorities would need to adopt a more preventive posture to potential shocks that could turn aside from the inflation target. In this context the development of models capable to describe and analysis the current policy objective is crucial.

When analyzing monetary policy objectives in open economies, the role of objective variables like exchange rate, inflation and output gap needs to have a special consideration. This work addresses the importance of these variables in a small open economy setting. It also implies a distinction between the consumer price index and the price index inflation. The small open economy framework can be used to assess the implications of alternative monetary policy rules for an open economy.

This paper develops an open economy extension of the basic New Keynesian model of a small open economy model as a limiting case of a two country dynamic general equilibrium framework featuring monopolistic competition and price stickiness. Moreover, the framework assumes no trade frictions and perfect capital markets. The structure presented includes the small open economy setting developed by De Paoli (2006) and Gali (2007). This work allows analyzing the case of two policy rules for central banks: targeting and Taylor-type regimes. Moreover, study what is the measure of inflation that the monetary authority should seek to stabilize. Finally, this framework is used to determine the implications of the alternative rules and compared them to the Taylor rule for Paraguay.

This document also illustrates some of the issues that emerge in the analysis of the different rules. When comparing different targeting rules, inflation targeting is the preferred policy for low levels of variances. When analyzing Taylor rules, producer price index inflation is the ideal policy rule for the 4 different calibrations proposed. When looking to the Taylor rule for Paraguay, the smallest variances values are attained when the monetary authority responds with an anti-inflation plan. Our results suggest that including the inflation as part of the stabilization goals of monetary policy can be welfare improving, in terms of little variances, from a small open economy point of view.

The paper is structured as follows. Section 2 introduces the model and derives a simple representation of the small open economy equilibrium dynamics. Section 3 is dedicated to the derivation of the monetary policy rules. Section 3.1 assesses the merits of three Targeting Rules: classical, inflation and exchange rate target. Section 3.2 discusses three different Taylor-type rules, a classical Taylor rule and two policies that fully stabilize CPI and PPI inflation, respectively. Section 4 discuss the Monetary Policy of Paraguay and derive a Taylor rule for the economy. Finally, to enrich the study, section 5 contains a numerical analysis of the model and presents impulse responses and variance of the target variables, as a measure of welfare losses. Section 6 includes concluding remarks.

## 2. The Model

The model consists of a two-country dynamic general equilibrium model with complete asset markets. Deviations from purchasing power parity arise from the existence of home bias in consumption. The dimension of this bias depends on the degree of openness and the relative size of the economy. We characterize the small open economy by taking the limit of the home size to zero. Prior to applying the limit, we derive the optimal equilibrium conditions for the general two country model. After the limit is taken, the two countries Home and Foreign, represent the small open economy and the rest of the world, respectively<sup>1</sup>.

Monopolistic competition and sticky prices are introduced in the small open economy in order to address issues of monetary policy. We further assume that home price setting follows a Calvo-type contract, which

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<sup>1</sup> Bianca De Paoli, "Monetary Policy and Welfare in a Small Open Economy", (2006).

introduces richer dynamic effects of monetary policy than a setup where prices are set one period in advance. Moreover, we abstract from monetary frictions by considering a cashless economy, as in Woodford (2003).

## 2.1 Preferences

We consider two countries, H (Home) and F (Foreign). The world economy is populated with a continuum of agents of unit mass, where the population in the segment  $[0; n]$  belongs to country H and the population in the segment  $(n; 1]$  belongs to country F. The utility function of a consumer  $j$  in country H is given by:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_s(h), \varepsilon_{Y,s})]$$

Households obtain utility from consumption  $U(C)$  and contribute to the production of a differentiated good  $y(h)$  attaining disutility  $V(y(h), \varepsilon_{Y,t})$ . Productivity shocks are denoted by  $\varepsilon_{Y,t}$ .

The isoelastic functional forms assumed for utility from consumption and disutility from production are:

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho} \text{ and } V(y_t, \varepsilon_{Y,t}) = \frac{\varepsilon_{Y,t}^{-\eta} y_t^{1+\eta}}{1+\eta}.$$

$C$  is a Dixit-Stiglitz aggregator of home and foreign goods as:

$$C = \left[ v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

where  $\theta > 0$  is the intratemporal elasticity of substitution and  $C_H$  and  $C_F$  are the two consumption sub-indices that refer to the consumption of home-produced and foreign-produced goods, respectively. The parameter determining home consumers' preference for foreign goods,  $(1-v)$  is a function of the relative size of the foreign economy,  $(1-n)$ ; and of the degree of openness,  $\lambda$ ; more specifically,  $(1-v) = (1-n)$ .

Similar preferences are specified for the rest of the world:

$$C = \left[ v^*{}^{\frac{1}{\theta}} C_H^*{}^{\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^*{}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

with  $v^* = n\lambda$ . That is, foreign consumers' preferences for home goods depend on the relative size of the home economy and the degree of openness. The specification of  $v$  and  $v^*$  generates a home bias in consumption as in Sutherland (2002).

The sub-indices  $C_H(C_H^*)$  and  $C_F(C_F^*)$  are Home (Foreign) consumption of the differentiated products produced in countries H and F. These are defined as follows:

$$C_H = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad C_F = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}$$

$$C_H^* = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad C_F^* = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}$$

where  $\sigma > 0$  is the elasticity of substitution across the differentiated products. The consumption-based price indices that correspond to the above specifications of preferences are given by:

$$P = [vP_H^{1-\theta} + (1-v)(P_F)^{1-\theta}]^{\frac{1}{1-\theta}}, \quad \theta > 0$$

$$P^* = [v^*P_H^{*1-\theta} + (1-v^*)(P_F^*)^{1-\theta}]^{\frac{1}{1-\theta}}, \quad \theta > 0$$

where  $P_H$  ( $P_H^*$ ) is the price sub-index for home-produced goods expressed in the domestic (foreign) currency and  $P_F$  ( $P_F^*$ ) is the price sub-index for foreign produced goods expressed in the domestic (foreign) currency.

$$P_H = \left[ \left( \frac{1}{n} \right) \int_0^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} \quad P_F = \left[ \left( \frac{1}{1-n} \right) \int_n^1 p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}$$

$$P_H^* = \left[ \left( \frac{1}{n} \right) \int_0^n p^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}} \quad P_F^* = \left[ \left( \frac{1}{1-n} \right) \int_n^1 p^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}$$

Moreover, we assume that the law of one price holds, therefore:  $p(h) = Sp^*(h)$  and  $p(f) = Sp^*(f)$  where the nominal exchange rate  $S_t$  denotes the price of foreign currency in terms of domestic currency. Therefore,  $P_H = SP_H^*$  and  $P_F = SP_F^*$ . However, the home bias specification leads to deviations from purchasing power parity, that is,  $P \neq SP^*$ . For this reason, we define the real exchange rate as  $Q = \frac{SP^*}{P}$ .

From consumers' preferences, we can derive the total demand for a generic good  $h$ , produced in country H, and the demand for a good  $f$ , produced in country F:

$$y^d(h) = \left[ \frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[ \frac{P_H}{P} \right]^{-\theta} \left[ vC + \frac{v^*(1-n)}{n} \left( \frac{1}{Q} \right)^{-\theta} C^* \right] \right\}$$

$$y^d(f) = \left[ \frac{p(f)}{P_F} \right]^{-\sigma} \left\{ \left[ \frac{P_F}{P} \right]^{-\theta} \left[ \frac{(1-v)n}{n} C + (1-v^*) \left( \frac{1}{Q} \right)^{-\theta} C^* \right] \right\}$$

Finally, to portray our small open economy we use the definition of  $v$  and  $v^*$  and take the limit for  $n \rightarrow 0$ . Consequently, the demand for a generic good  $h$ , produced in country H, and the demand for a good  $f$ , produced in country F can be rewritten as:

$$y^d(h) = \left[ \frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[ \frac{P_H}{P} \right]^{-\theta} \left[ (1-\lambda)C + \lambda \left( \frac{1}{Q} \right)^{-\theta} C^* \right] \right\}$$

$$y^d(f) = \left[ \frac{p^*(f)}{P_F^*} \right]^{-\sigma} \left\{ \left[ \frac{P_F^*}{P^*} \right]^{-\theta} C^* \right\}$$

These equations show how external changes in consumption affect the small open economy, but the reverse is not true. Moreover, movements in the real exchange rate do not modify the rest of the world's demand.

## 2.2 The asset market structure

We assume that markets are complete domestically and internationally. In each period  $t$  the economy faces one of the infinitely many events  $s^t \in Y$  (where  $Y$  is the set of infinitely many states). We denote the history of events up to and including period  $t$  by  $x^t$ . Looking ahead from period  $t$ , the conditional probability of occurrence of state  $s^{t+1}$  is  $u(s^{t+1} | x^t)$ . The initial realization  $s^0$  is given. We represent the asset structure by having complete contingent one period nominal bonds denominated in the home currency. We let  $B^j(s^{t+1})$  denote the home consumer's holdings of this bond, which pays one unit of the home currency if state  $s^{t+1}$  occurs and 0 otherwise, and we let  $Q(s^{t+1} | x^t)$  denote the price of one unit of such a bond at date  $t$  and state  $s^t$  in units of domestic currency. Therefore, consumer  $j$  faces a sequence of budget constraints given by:

$$P(s^t)C^j(s^t) + \sum_{s^{t+1} \in Y} Q(s^{t+1} | x^t) \beta^j(s^{t+1}) \leq \beta^j(s^t) + (1 - \tau_t)p^j(s^t)y^j(s^t)dh + P_H(s^t)Tr(s^t)$$

A similar expression can be derived for the foreign economy. Households at home maximize the utility function subject to the sequences of budget constraints and their optimal allocation of wealth across the different state contingent bonds implies that:

$$Q(s^{t+1} | x^t) = \beta u(s^{t+1} | x^t) \frac{U_c(C(s^{t+1}))}{U_c(C(s^t))} \frac{P(s^t)}{P(s^{t+1})}$$

Similarly for the foreign economy:

$$Q(s^{t+1} | x^t) = \beta u(s^{t+1} | x^t) \frac{U_c(C^*(s^{t+1}))}{U_c(C^*(s^t))} \frac{S(s^t)P^*(s^t)}{S(s^{t+1})P^*(s^{t+1})}$$

Therefore, the optimal risk sharing setting implies that the intertemporal marginal rate of substitution (in nominal terms) is equalized across countries.

$$\frac{U_c(C_{t+1}^*)}{U_c(C_t^*)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{S_{t+1}P_t}{S_tP_{t+1}}$$

This specification for the asset market implies that the risk arising from movements in agent's nominal wealth is shared with the rest of the world. However, because of deviations from purchasing power parity, real exchange rate movements may lead to differences between home and foreign real income and, consequently, differences in the evolution of consumption across borders.

### 2.3 Price-setting Mechanism

Prices follow a partial adjustment rule a la Calvo (1983). Producers of differentiated goods know the form of their individual demand functions, and maximize profits taking the overall market prices and products as given. In each period a fraction  $\alpha \in [0,1)$  of randomly chosen producers is not allowed to change the nominal price of the good it produces. The remaining fraction of firms, given by  $(1 - \alpha)$ , chooses prices optimally by maximizing the expected discounted value of profits. Therefore, the optimal choice of producers that can set their price  $\tilde{p}_j(j)$  at time  $T$  is:

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left( \frac{\tilde{p}_j(j)}{P_{H,t}} \right)^{-\sigma} Y_{H,T} \left[ \frac{\tilde{p}_j(j) P_{H,t}}{P_{H,t} P_T} - \frac{\sigma}{(1 - \tau_T)(\sigma - 1)} \frac{V_y(\tilde{y}_{t,T}(j), \epsilon_{Y,T})}{U_c(C_T)} \right] \right\} = 0$$

Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production, represented by  $\frac{\sigma}{(1 - \tau_t)(\sigma - 1)}$ . We allow for fluctuations on this wedge by

assuming a time varying proportional tax  $\tau_t$ . Hereafter, we refer to these fluctuations as mark up shocks  $u_t$  where  $u_t = \frac{\sigma}{(1-\tau_t)(\sigma-1)}$ .<sup>2</sup>

Given the Calvo-type setup, the price index evolves according to the following law of motion:

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)(\tilde{p}_t(h))^{1-\sigma}$$

The rest of the world has an analogous price setting mechanism.

## 2.4 A log-linear representation of the model

In this section we present the model's equilibrium conditions in log-linear form, while Appendix A contains the derivation of the model.

The model is approximated around a steady state in which the exogenous variables  $(\varepsilon_{y,t}, u_t)$  assume the values  $(\bar{\varepsilon}_{y,t} \geq 0, u \geq 1)$  and producer price inflation is set as  $\bar{\pi}_t \equiv P_t/P_{t-1} = 1$ . In addition, in this steady-state  $\bar{q}_t = 1$ ,  $\bar{c}_t = 1$ ,  $\bar{c}_t = \bar{c}_t^*$ ,  $\bar{\pi}_t = \bar{\pi}_t^*$  and  $\bar{y} = \bar{y}^*$ . Log deviations from the steady state are denoted with a hat.

**Table 1: Small Open Economy's system of equilibrium conditions**

$\hat{\pi}_t = k(\rho\hat{c}_t + \eta\hat{y}_t + \lambda(1-\lambda)^{-1}\hat{q}_t + \hat{\mu}_t - \eta\hat{\varepsilon}_t) + \beta E_t \hat{\pi}_{t+1}$	Home Aggregate Supply
$\hat{y}_t = (1-\lambda)\hat{c}_t + \lambda\hat{c}_t^* + Y\hat{q}_t$	Home Aggregate Demand
$\hat{c}_t = \hat{c}_t^* + (1/\rho)\hat{q}_t$	Risk Sharing

As shown in Table 1, the small open economy's log-linearized equilibrium dynamics can be summarized by an Aggregate Supply, an Aggregate Demand and a Risk Sharing Condition. The first equation represents the small open economy's Phillips curve, the second illustrates how the demand for the small open economy's products depends on foreign and domestic consumption and, the third equation is derived from the complete market assumption, and represents the optimal risk sharing agreement between agents in the small economy and the rest of the world. We define  $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$  and  $Y = \frac{\theta\lambda(2-\lambda)}{1-\lambda}$ .

The variables  $\hat{c}_t$  and  $\hat{c}_t^*$  denote domestic and foreign consumption,  $\hat{y}_t$  denotes domestic output,  $\hat{q}_t$  denotes the real exchange rate and  $\hat{\pi}_t \equiv \ln(P_{H,t}/P_{H,t-1})$  denotes the producer price inflation. The stochastic environment is characterized by two domestic structural shocks (mark-up shocks,  $\hat{\mu}_t$  and productivity shocks,  $\hat{\varepsilon}_t$ ) and an external conditions  $\hat{c}_t^*$ .

Given the above exogenous variables, the small open economy system of equilibrium conditions is closed by specifying the monetary policy rule. We consider the case in which monetary policy follows a targeting rule. Therefore, an explicit expression for the evolution of the monetary policy instrument (i.e. the nominal interest rate) is not specified. Following this rule, the central bank stabilizes movements in the target variables in order to implement the most efficient allocation of resources. Within targeting rules, we analyze the performance of a classical targeting rule (considered as a benchmark), a domestic inflation targeting and an exchange rate targeting.

Moreover, apart from analyzing targeting rules, we also consider the case in which the central bank follows standard policy rules. In particular, we analyze the performance of a classical Taylor type policy rule (used as a benchmark), a producer price index (PPI) inflation policy rule and a consumer price index (CPI) inflation policy rule. Finally, we compare the last three policy rules with a Taylor rule for the Paraguayan economy.

<sup>2</sup> In this model we assume  $u = \frac{1}{(1-\lambda)}$ . This mean that the steady state level of output ought to be efficient from the small open economy's point of view.



**Table 2: Foreign equilibrium conditions**

$\widehat{\Pi}_t^* = k(\rho\hat{c}_t^* + \eta\hat{y}_t^* + \hat{\mu}_t^* - \eta\hat{\varepsilon}_t^*) + \beta E_t \widehat{\Pi}_{t+1}^*$	Foreign Aggregate Supply
$\hat{y}_t^* = \hat{c}_t^*$	Foreign Aggregate Demand

As shown in Table 2, foreign dynamics are governed by foreign supply and demand conditions. The specification of the foreign policy rule completes the system of equilibrium conditions which determine the evolution of foreign inflation  $\widehat{\Pi}_t^*$ , foreign output  $\hat{y}_t^*$  and foreign consumption  $\hat{c}_t^*$ . The economic dynamics of the rest of the world are independent of the dynamics of the real exchange rate or any variable in small open economy. Therefore, the policymaker of the small open economy can treat  $\hat{c}_t^*$  as exogenous. The policy choice of the rest of the world may modify the way in which foreign structural shocks affect  $\hat{c}_t^*$ , but it does not influence how the latter affects the small open economy.

### 3. Monetary Policy Rules

#### 3.1 Targeting Rules

In this section we analyze the case in which the monetary policy objective is represented in the form of a targeting rule. Given the exogenous variables and the system of equilibrium conditions of the small open economy and the rest of the world, the model is closed by specifying this monetary policy rule.

First, we consider a classical targeting rule of the form:<sup>3</sup>

$$(1 + l)\Phi_y\hat{y}_t + \rho(1 - \lambda)\Phi_q\hat{q}_t + (\rho + \eta(1 + l))\kappa\Phi_\Pi\widehat{\Pi}_t = 0$$

where  $l = (\rho\theta - 1)\lambda(2 - \lambda)$ . The weights of inflation, output and exchange rate  $\Phi_\Pi$ ,  $\Phi_y$  and  $\Phi_q$  depend on the structural parameters of the model. The expressions for these variables are specified in Appendix B.

The above expression prescribes responding to movements in inflation, output and the real exchange rate. When following this policy rule, the central bank may allow some variability in inflation in order to respond to costly movements in other variables.

Moreover, we consider two special cases of targeting regimes. The first is a full inflation targeting rule and the second is a full exchange rate targeting rule. Both policies are specified according to the following expression:

$$\Phi_y\hat{y}_t + \Phi_q\hat{q}_t + \Phi_\Pi\widehat{\Pi}_t = 0$$

Each one of these rules is specified by alternative configurations for  $\Phi_\Pi$ ,  $\Phi_y$  and  $\Phi_q$  according to the target they follow. For inflation targeting, we assume a very aggressive anti-inflation stance for the inflation weight and very low responses to output and exchange rate fluctuations. When targeting the exchange rate we assume a sufficiently large  $\hat{q}_t$  accordingly to our strong stabilization purpose.

In Section 5 we show the impulse responses to domestic productivity and foreign shocks under the above systems. Furthermore, we illustrate the contribution to welfare losses, in terms of variance of inflation, output and real exchange rate, given these 3 targeting rules and a calibration of the model's parameters.

#### 3.2 Taylor Rules

<sup>3</sup> This rule is an adjusted version of the Optimal Targeting Rule of Bianca De Paoli, "Monetary Policy and Welfare in a Small Open Economy" (2006).

This section studies the case in which the monetary policy objective is represented in the form of a monetary policy instrument: the nominal interest rate. The policies considered are stylized Taylor-type rules. So, given the system of equilibrium conditions of the small open economy and the rest of the world, the model is closed with the Taylor rule. Specifically, we study a classical, PPI and CPI inflation Taylor-type policy rule and a “Paraguayan” Taylor rule.

In order to retrieve the value of the nominal interest rate we use the households’ intertemporal choice (i.e. the Euler equation). So, the stochastic Euler equation is<sup>4</sup>:

$$Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\}$$

where  $Q_t \equiv E_t \{Q_{t,t+1}\}$  denotes the price of a one-period discount bond paying off one unit of domestic currency in  $t + 1$ .

This equation can be written in log-linearized form as:

$$\hat{c}_t = E_t \{ \hat{c}_{t+1} \} - \frac{1}{\rho} (i_t - E_t \hat{\Pi} CPI_{t+1} - p)$$

where  $i_t \equiv -Q_t \log$  is the short term nominal rate,  $p \equiv -\log(\beta)$  is the time discount rate, and  $\hat{\Pi} CPI_t$  is Consumer Price Index (CPI) Inflation.

The above equation can be rewritten as:

$$\hat{c}_t = E_t \{ \hat{c}_{t+1} \} - \frac{1}{\rho} (i_t - E_t \hat{\Pi} CPI_{t+1} + \log \beta)$$

Now, we need a connection between PPI and CPI Inflation. Domestic inflation (PPI), defined as  $\hat{\Pi}_t \equiv \ln(P_{H,t}/P_{H,t-1})$  and CPI inflation are linked according to the relation<sup>5</sup>:

$$\hat{\Pi} CPI_t = \hat{\Pi}_t + \lambda \Delta(\hat{q}_t)$$

The gap between the two measures of inflation is proportional to the percent change in the terms of trade, with the coefficient of proportionality given by the openness index  $\lambda$ . Next, a relationship is derived between the terms of trade and the real exchange rate:

$$q_t = \int_0^1 (e_{i,t} + p_t^i - p_t) di = e_t + p_t^* - p_t = s_t + p_{H,t} + p_t$$

$$q_t = (1 - \lambda) s_t$$

where  $s_t$  is terms of trade and  $e_t$  the effective nominal exchange rate.

Combining the previous equations yields the relation for CPI and PPI inflation:

$$\hat{\Pi} CPI_t = \hat{\Pi}_t + \left( \frac{\lambda}{1-\lambda} \right) \hat{q}_{t+1} - \left( \frac{\lambda}{1-\lambda} \right) \hat{q}_t$$

With these two equations we are ready to express the monetary policy objective in the form of a Taylor rule.

First, we consider a classical Taylor type interest rate rule of the form<sup>6</sup>:

<sup>4</sup> Jordi Gali, “Monetary Policy, Inflation and Business Cycle”, (2007).

<sup>5</sup> Idem.

<sup>6</sup> Consistent with Taylor(1993).

$$i_t = p + \Phi_{\Pi} \widehat{\Pi}_t + \Phi_y \widehat{y}_t$$

where  $p \equiv -\log(\beta)$  is consistent with a zero inflation steady state.

The inflation and output weights,  $\Phi_{\Pi}$  and  $\Phi_y$ , are non-negative coefficients determined by the central bank, that describe the strength of the interest rate response to deviations of inflation or the output gap from their target levels.

Under the assumption of non-negative values for  $(\Phi_{\Pi}, \Phi_y)$ , a necessary and sufficient condition for the equilibrium to be unique, is given by<sup>7</sup>

$$\left[ \left( \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \right) \left( \frac{(1-\lambda)}{(1+\lambda)\sigma} \right) \left( p + \frac{\eta+\lambda}{1-\lambda} \right) \right] (\Phi_{\Pi} - 1) + (1-\beta)\Phi_y > 0$$

In other words, the monetary authority should respond to deviations of inflation and the output gap from their target levels by adjusting the nominal rate with “sufficient strength”. So, the inflation and output weights in the Taylor rule have to satisfy this determinacy condition.

We also study a PPI and CPI inflation Taylor rules:

$$i_t = p + \Phi_{\Pi} \widehat{\Pi}_t$$

$$i_t = p + \Phi_{\Pi} \widehat{\Pi} CPI_t$$

where  $p \equiv -\log(\beta)$  is consistent with a zero inflation steady state. In the first policy, the domestic interest rate responds systematically to domestic inflation (PPI), whereas the second assumes that CPI inflation is the variable the central bank reacts to.

In Section 5 we show the responses of several macroeconomic variables to different shocks under the above rules for a calibrated version of the economy. Furthermore, we compare the contribution to welfare losses, in terms of variance of inflation and output, of the original Taylor’ values<sup>8</sup> of  $\Phi_{\Pi}$  and  $\Phi_y$  to those associated with an aggressive anti-inflation stance and a strong stabilization of output gap.

## 4. Monetary Policy in Paraguay

### 4.1 Characteristics of the Monetary Policy in Paraguay<sup>9</sup>

The monetary environment in Paraguay is characterized by intermediate dollarization, high volatility, and a weak banking system. Regarding to inflation, Paraguay is one of the countries in Latin-American that have never experienced three-digit inflation over the past 50 years<sup>10</sup>. However, inflation over the past 10 years has been on average higher than in most other countries in the region. Moreover, over the recent past, inflation has also been highly volatile. The volatility reflects external factors like the location of Paraguay as an exporter of primary commodities and the world increase of food and energy prices. However, the domestic component to inflation volatility cannot be underestimated<sup>11</sup>.

One factor which certainly damages the credibility of monetary policy in Paraguay is the multiplicity of objectives. While the charter of the Central Bank of Paraguay (Banco Central de Paraguay, BCP)

<sup>7</sup> See Galí (2007) for a discussion.

<sup>8</sup> This is calibrated version of the Taylor rule as Taylor (1993).

<sup>9</sup> I thank Santiago Peña and Bernardo Rojas for suggestions in this explanation.

<sup>10</sup> The other country is Colombia

<sup>11</sup> A document of the Monetary International Fund (2009) suggest that in Paraguay monetary factors, in particular currency in circulation, have played a major role in determining long-run inflation, whereas foreign prices, in particular from Brazil, and some food products have had a large impact on the short-term dynamics of inflation.

emphatically states that the primary objective of the BCP is to ensure price stability and financial system, in practice the pressure to control cyclical fluctuations of the exchange rate and output variation is very high. In this contradictory context, it is very difficult for the central bank to exercise an effectively control to the inflation.

Recognizing the necessity of improving the current monetary policy, the authorities of the BCP, have begun to outline strategies to be migrating the current monetary policy regime to an inflation targeting regime. During the past years, the BCP has developed macroeconomics model to describe the transmission mechanism of monetary policy. The BCP has also announced an end-year target for inflation with a broad band of  $\pm 2.5$  percentage points. This way of making monetary policy is consistent with a gradual response of the interest rate to shocks affecting inflation and allow a gradually convergence of inflation to its target.

## 4.2 Paraguayan Taylor Rule

To illustrate the factors influencing monetary policy, we estimate a Taylor rule, and compare the results for Paraguay with those of classical, PPI and CPI inflation Taylor Rule.

To analyze the monetary policy rule in Paraguay, we use the interest rate on central bank bills called *letras de regulación monetaria* (LRMs) as the policy interest rate. The interest rate on LRMs is closely correlated with other relevant interest rates, such as the deposit rate on local currency and to the lending rate in local currency. The measure of headline inflation used by the BCP is the Consumer Price Index (CPI)<sup>12</sup>.

The equation that describes the policy rule of the Paraguayan monetary authority can be represented as following:

$$lpm_t = \beta_1 lpm_{t-1} + \beta_2 (\hat{\Pi}_t - \bar{\Pi}) + (1 - \beta_2) (\hat{y}_t - \bar{y}) + \varepsilon_t$$

This loss function illustrates that the nominal interest rate on central bank bills (*letras de regulación monetaria*),  $lpm_t$  is a function of its own lag and the inflation and output gap, where  $\varepsilon_t$  is the residue of the Taylor rule.

Finally, we can rewrite the above policy rule taking into account that the nominal interest rate on BCP bills is analogous with the short term nominal interest rate and that the inflation in the country is measured by CPI. The policy rule equation with these specifications can be represented as follows:

$$i_t = (1 - \varsigma)p + \varsigma i_{t-1} + \varsigma i_t + \varsigma \Phi_{\Pi} \hat{\Pi} CPI_t + \varsigma \Phi_y \hat{y}_t$$

where  $p \equiv -\log(\beta)$  is the time discount rate consistent with a zero inflation steady state. The parameter  $\varsigma$  is consistent with the results found by Santos and Monfort (2009) for a Taylor Rule for Paraguay. In this policy rule the nominal interest rate is a function of its own lag, which reflects the idea of smoothing the path of the policy interest rate, the output and inflation gap.

So, in the specific case of the Paraguayan monetary policy, the model is closed with system of equilibrium conditions of the small open economy and the rest of the world, and by specifying the above Taylor rule for Paraguay. Later on, we compare the results for Paraguay's Taylor rule with those of classical, PPI and CPI Inflation Taylor Rule.

## 5. A Numerical Analysis of Alternative Rules

<sup>12</sup> Inflation until 2007 was measured by a CPI based on the 1992 household budget. Since January 2008, the BCP started publishing a new CPI based on the 2005 household budget survey. This new basket gives less weight to food items, which have been one of the main reasons of price increases over the past two years.

## 5.1 Calibration

This section presents some quantitative results based on a calibrated version of the small open economy. In our baseline calibration we assume a unitary elasticity of intertemporal substitution, i.e.  $\rho = 1$ . We suppose a labor supply elasticity of  $\eta = 0.47$  following Rotemberg and Woodford (1997). Furthermore, the elasticity of substitution between home and foreign goods,  $\theta$ , is assumed to be 3<sup>13</sup>. Parameter  $\alpha$  is set equal to 0.66, a value consistent with an average length of price contract of 3 quarters. Moreover, the elasticity of substitution between differentiated goods,  $\sigma$ , is assumed to be 10 as in Benigno and Benigno (2003). It is assumed that  $\beta = 0.99$ , which implies a riskless annual return of about 4 percent in the steady state. The degree of openness  $\lambda$  is set at 0.4. The latter corresponds to the import/GDP ratio in Paraguay. The calibration of the interest rate rules follows the original Taylor calibration and sets  $\Phi_{\Pi}$  equal to 1.5.

In order to calibrate the stochastic properties of the exogenous driving forces, let us fit AR(1) processes with persistence  $\kappa^{(*)}$  and standard deviation  $sdv^{(*)}$ . The estimates are described in Table 3. The standard deviations of this process are described in Table 4.

In order to compute the model we use the MATLAB program "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function" by Stephanie Schmitt-Grohe and Martin Uribe (November 19, 2009).<sup>14</sup>

**Table 3: Process of the exogenous variables**

$\hat{\varepsilon}_t = \kappa^{(\varepsilon)} \hat{\varepsilon}_{t-1} + \varepsilon_t^{\varepsilon}$	Productivity Shock of the Small Open Economy
$\hat{\varepsilon}_t^* = \kappa^{(\varepsilon^*)} \hat{\varepsilon}_{t-1}^* + \varepsilon_t^{\varepsilon^*}$	Productivity Shock of the Foreign Economy
$\hat{c}_t^* = \kappa^{(c^*)} \hat{c}_{t-1}^* + c_t^*$	Foreign Consumption Shock
$\hat{u}_t = \kappa^{(u)} \hat{u}_{t-1} + \varepsilon_t^u$	Mark up Shock of the Small Open Economy
$\hat{u}_t^* = \kappa^{(u^*)} \hat{u}_{t-1}^* + u_t^*$	Mark up Shock of the Foreign Economy

A complete description of the parameters of the model is shown in Table 4. This table describes the parameter values used in the quantitative analysis. The values of the inflation, output and exchange rate weights satisfy the determinacy condition.

**Table 4: Parameters of the model**

Model Parameters		Value 1*	Notes	Value 2**	Notes
$\lambda$	Degree of openness of the economy	0.4	Implies a 40% import share of the GDP (Paraguay). Consistent with Gali (2007).	-	$1/2, 1/3, 1/4, \text{ and } 1/5$ To measure the contribution to welfare losses of the Paraguayan Taylor rule.
$\theta$	Elasticity of substitution between home and foreign goods	3	-	1	Following Gali (2007). 1, 2, 4, 5 and 6 To measure the contribution to welfare losses of the Paraguayan Taylor rule.
$\rho$	Intertemporal elasticity of substitution	1	-	2, 3, 4, 5 and 6	To measure the contribution to welfare losses of the Paraguayan Taylor rule.
$\eta$	Inverse of the elasticity of labor production	0.47	Following Rotemberg and Woodford (1997)	1/3	Following Gali (2007)
$\alpha$	Degree of price rigidity	0.66	Characterizing an average length of price contract of 3 quarters	0.75	Average period of 1 year between price adjustments. Following Gali (2007)
$\beta$	Subjective discount factor	0.99	Specifying a quarterly model with 4% steady-state real interest rate	-	-
$\sigma$	Elasticity of substitution across differentiated products	10	Following Benigno and Woodford (2005)	6	Following Gali (2007)
$\zeta$	Parameter of the Paraguayan Taylor Rule	0.8	Consistent with Santos and Monfort (2009)	-	-
$\kappa$	$(1 - \alpha\beta)(1 - \alpha)/\alpha(1 + \sigma\eta)$	-	-	-	-
$\Upsilon$	$\theta\lambda(2 - \lambda)/1 - \lambda$	-	-	-	-
$\Phi_{\Pi}$	Weight on Inflation	1.5	Consistent with Taylor (1993)	5	To measure the contribution to welfare losses of

<sup>13</sup> This leads to a specification where Home and Foreign goods are substitutes in the utility, given that  $\rho\theta > 0$ .

<sup>14</sup> Specifically in this program we use the neoclassical growth example.

					Taylor rules. In targeting rules this value is a function of the structural parameters
$\Phi_y$	Weight on Output	0.125	Consistent with Taylor (1993)	0 and 1	To measure the contribution to welfare losses of Taylor rules. In targeting rules this value is a function of the structural parameters
$\Phi_q$	Weight of Real Exchange Rate	-	-	-	In targeting rules this value is a function of the structural parameters
$sdv(\hat{\varepsilon}_t)$	Standard deviation of home productivity shock	0.0071	Consistent with Gali and Monacelli (2005) and Kehoe and Perri (2002)	-	-
$sdv(\hat{\mu}_t)$	Standard deviation of home mark up shock	0.0013	Consistent with Adolfson et al. (2007) and Smets and Wouters (2003)	-	-
$sdv(\hat{\varepsilon}_t^*)$	Standard deviation of foreign consumption up shock	0.0129	Following Lubik and Schorfheide (2007)	-	-
$sdv(\hat{\varepsilon}_t^*)$	Standard deviation of foreign productivity shock	0.0013	Consistent with Gali and Monacelli (2005) and Kehoe and Perri (2002)	-	-
$sdv(\hat{\mu}_t^*)$	Standard deviation of foreign mark up shock	0.0013	Consistent with Adolfson et al. (2007) and Smets and Wouters (2003)	-	-
$\kappa(\varepsilon)$	Persistence of home productivity shock	0.66	Following Gali and Monacelli (2005)	-	-
$\kappa(u)$	Persistence of home mark up shock	0.99	Following Adolfson et al. (2007)	-	-
$\kappa(c^*)$	Persistence of foreign consumption shock	0.66	Following Gali and Monacelli (2005)	-	-
$\kappa(\varepsilon^*)$	Persistence of foreign productivity shock	0.66	Following Gali and Monacelli (2005)	-	-
$\kappa(u^*)$	Persistence of foreign mark up shock	0.99	Following Adolfson et al. (2007)	-	-

\*General values to calibrate Targeting and Policy Rules.

\*\*Values to calibrate Taylor type classical Rule.

## 5.2 Impulse Responses

Taking into account the above specification, we analyze how monetary policies respond to the different shocks. First we describe the dynamic effects of a domestic productivity and foreign shock on a number of macroeconomic variables under different regimes.

Figure 1 shows the impulse responses of consumption, output, the real exchange rate and domestic inflation following productivity and foreign shocks under a classical targeting rule. As we can see in Figure 1.1, under this regime, a higher productivity at home increases domestic consumption and output. In addition, a larger supply of domestic goods leads to a depreciation in the real exchange rate. The zero measure specification of the Home economy enables us to study how the monetary authority should respond to fluctuations on external conditions, when there are no feedback effects. Figure 1.2 presents the impulse response of the various domestic variables to a foreign shock. Given a unit of innovation in  $c_t^*$ , domestic consumption increases and there is a real exchange rate appreciation. The impact on domestic competitiveness leads to a fall in home production.

As illustrated in Figure 2.1, when the economy is under an Inflation targeting rule, the policy responses to a productivity shock implies similar results for consumption, output, real exchange rate and domestic inflation. However, the increase in consumption and output is higher under this policy. In addition, the depreciation in the real exchange rate and the impact on inflation is also larger. Figure 2.2 presents the impulse response to a foreign shock. Comparing to the first policy, the fall in home production is stronger and real exchange rate is appreciated with more force under this regime.

Figure 3 displays the impulse responses under an Exchange Rate targeting rule. A productivity shock leads to a higher response of home consumption and the depreciation in the real exchange rate is also higher under this regime, as shown in figure 3.1. The impulse responses to a foreign shock illustrates an increase in home output and a slightly higher impact on the domestic consumption, comparing to the other two targeting regimes. Furthermore the impact on inflation is much stronger, too. This results can be shown un Figure 3.2.

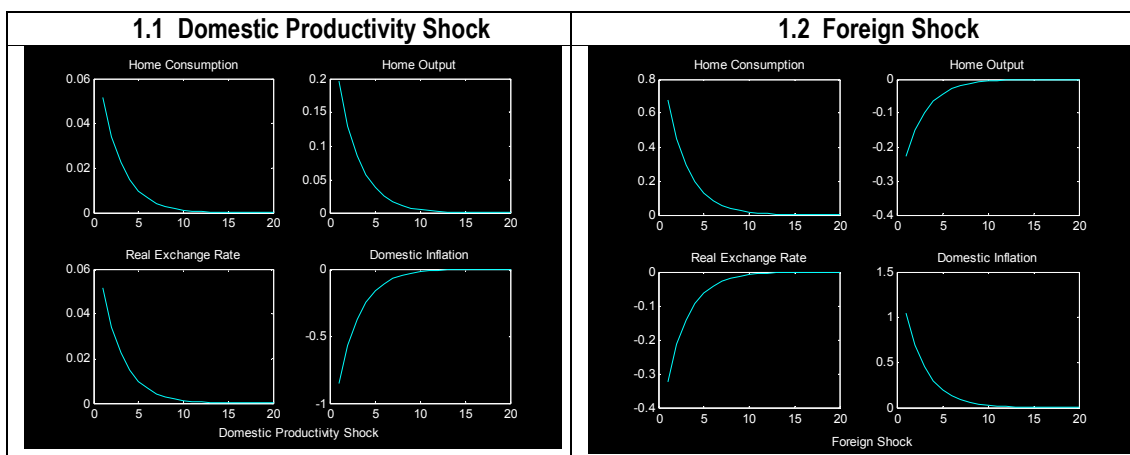
The following figures display the impulse responses under the different Taylor rules. Figure 4 explains the impact of to a unit innovation in  $\hat{\varepsilon}_t$  and  $c_t^*$  under a classical Taylor rule. The first shock leads to a reduction in the domestic interest rate, as it is needed in order to support the transitory expansion in consumption and output. The real exchange rate suffers a late depreciation. Notice that this rule generates a fall in both domestic and CPI Inflation. A unit innovation in  $c_t^*$ , generates an increase in domestic

consumption and a fall in output. Both inflation indexes suffer a reduction. The real exchange rate suffers a late appreciation and the nominal interest rate has the same response as in the case of a productivity shock. These results are exposed in Figure 4.1 and 4.2, respectively.

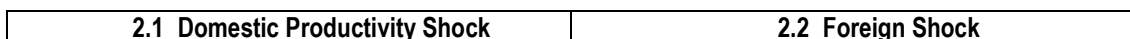
The dynamic behavior of the same variables under the two stylized Taylor rules (PPI and CPI) is illustrated in Figures 5 and 6. In Figures 5.1 and 6.1 we can see that the productivity shock leads to a transitory expansion in consumption and output given the persistent reduction in the domestic interest rate. Notice that both rules generate, unlike the benchmark policy, a permanent fall in both domestic and CPI prices. A key difference between these two Taylor rules concerns the behavior of the CPI Inflation. Under PPI there is a reduction on CPI Inflation reverting gradually to the steady state afterwards, while under CPI (mirroring closely the response under the classical Taylor rule), the initial response of the CPI Inflation is more muted and is followed by a hump-shape pattern. Looking to Figures 5.2 and 6.2 we can see the impact of the fluctuations on external conditions. These results do not change significantly comparing the classical Taylor Rule.

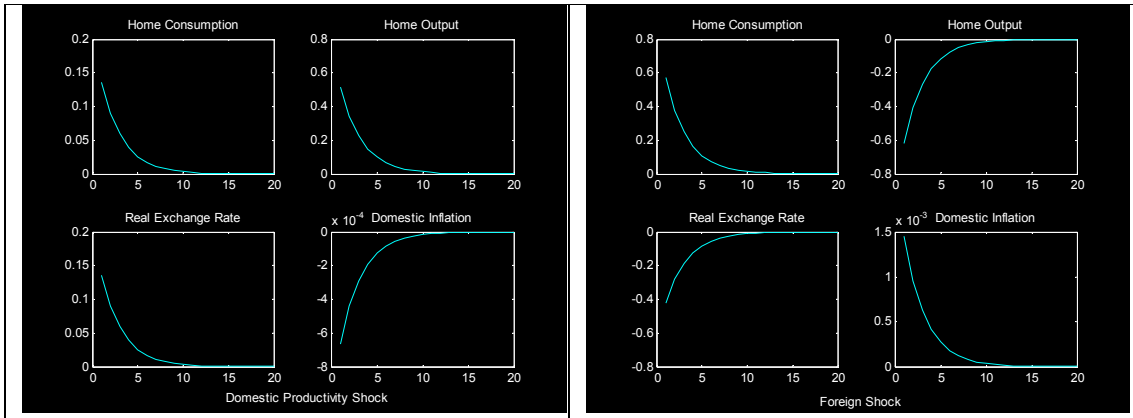
Finally, Figure 7 displays the dynamic behavior of the macroeconomic variables under the Taylor Rule for Paraguay. By looking these graphs, we can determine the direction, duration and magnitude of the effect of monetary policy rules on the Paraguayan economy. In Figure 7.1 we can see the impact of a productivity shock. A higher productivity leads to a transitory expansion in consumption and output given the persistent reduction in the nominal interest rate (on *letras de regulación monetaria*). The overall effect is dissipated around ten quarters. The impact on CPI Inflation is higher than the two Taylor rules (PPI and CPI). Regarding the exchange rate, a higher productivity depreciates the currency temporarily and the effect is dissipated approximately 12 quarters. A unit innovation in  $c_t^*$  generates an increase in domestic consumption and a fall in output. Regarding the CPI Inflation, a higher foreign consumption increase temporarily this index and the effect is dissipated approximately in 10 quarters. The real exchange rate suffers a late appreciation and the nominal interest rate has the same response as the case of a productivity shock. These results can be seen in Figure 7.2.

**Figure 1: Impulse responses to different shocks under a classical targeting rule**

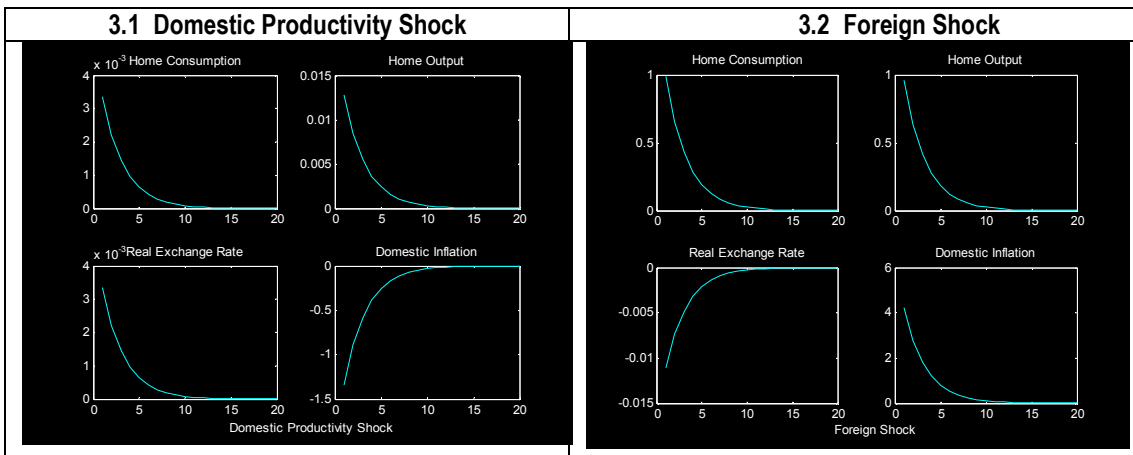


**Figure 2: Impulse responses to different shocks under an Inflation targeting rule**

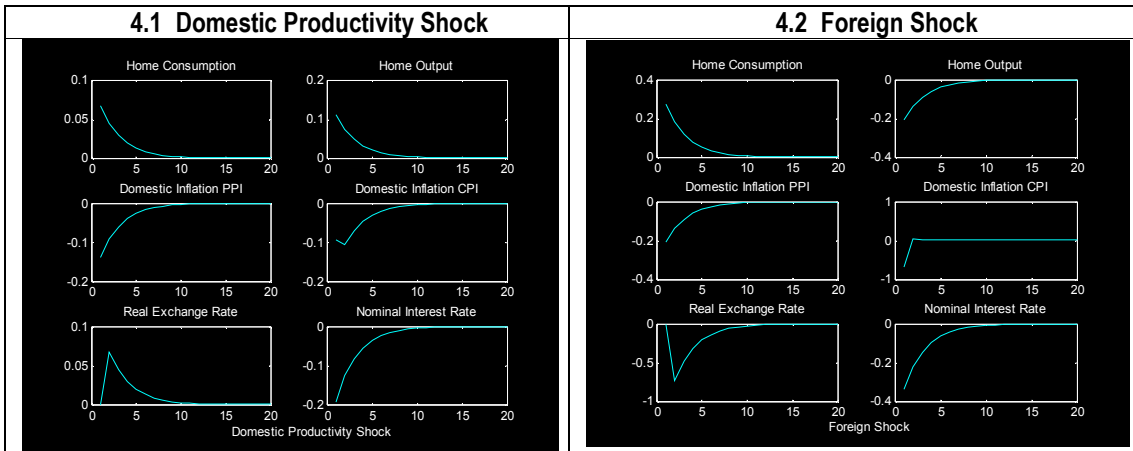




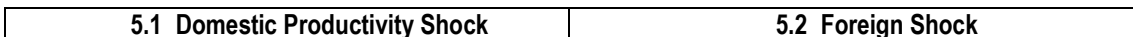
**Figure 3:** Impulse responses to different shocks under an Exchange Rate targeting rule



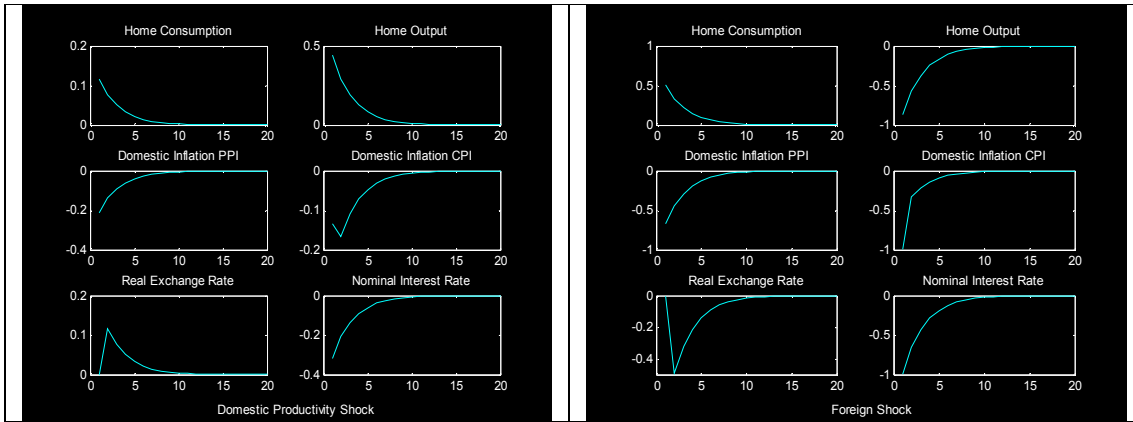
**Figure 4:** Impulse responses to different shocks under a classical Taylor type policy rule



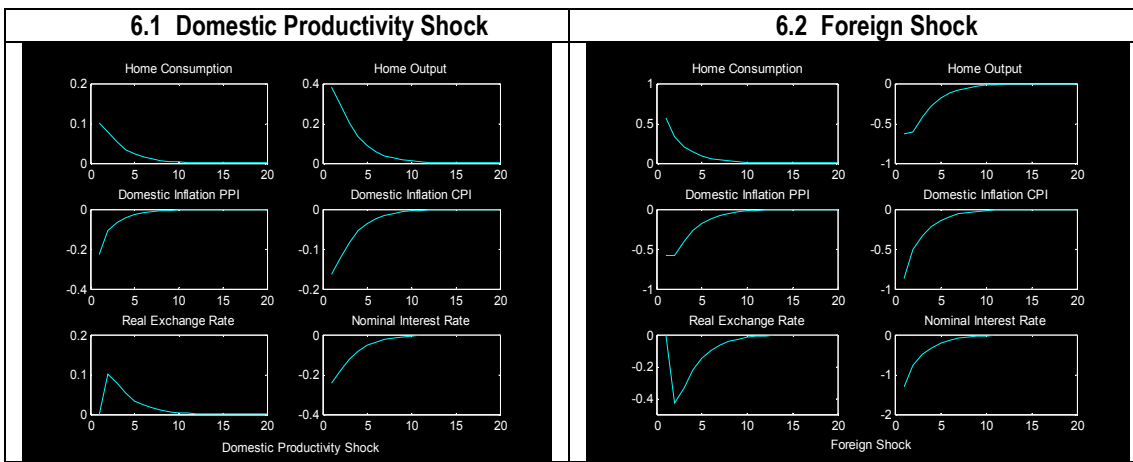
**Figure 5:** Impulse responses to different shocks under a Producer Price Index policy rule



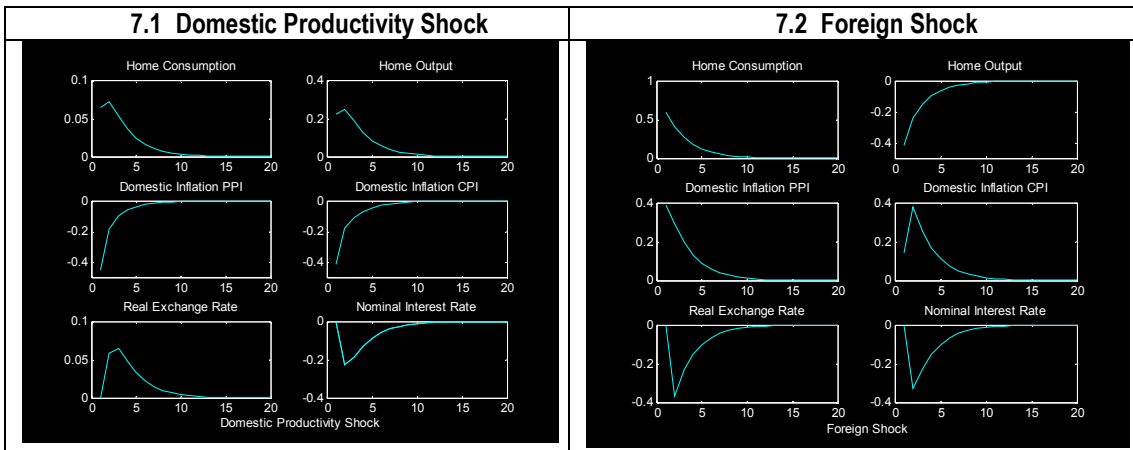




**Figure 6:** Impulse responses to different shocks under a Consumer Price Index policy rule



**Figure 7:** Impulse responses to different shocks under the Paraguayan policy rule



### 5.3 Variances of Inflation, Output gap and Exchange Rate

In order to complement the quantitative analysis, this section analyses the variance fluctuations of the objective variables of a specific policy rule. Given a policy rule and a calibration of the model's parameters, one can determine the implied variance of inflation, output gap and exchange rate that can be

approximate as a measure of welfare losses associated with that policy regime<sup>15</sup>. The unconditional variance-covariance matrixes of endogenous and exogenous variables were computed using the method of doubling algorithm. All entries were multiplied by 100 in order to make the values of the variance bigger. Therefore, they are to be read as per ten thousand units of steady state consumption.

Table 1 reports the variances of inflation, output gap and exchange rate associated with rules analyzed in the section 3.1: classical, inflation and exchange rate targeting rules. This panel reports the variances in the case of the benchmark parameterization. Under the calibration considered, an exchange rate targeting implies a substantially larger deviation than the classical and the inflation targeting rule. Domestic inflation targeting is the preferred policy rule for low levels of variances. However, the implied variances are quantitatively small for all policy regimes.

**Table 1: Variances of control variables under alternative targeting rules**

Classic Targeting Rule	Inflation Targeting	Exchange Rate Targeting
$\text{var}(\hat{y}_t) = 0.0105$	$\text{var}(\hat{y}_t) = 0.0238$	$\text{var}(\hat{y}_t) = 0.0245$
$\text{var}(\hat{\pi}_t) = 0.2017$	$\text{var}(\hat{\pi}_t) = 0.0000$	$\text{var}(\hat{\pi}_t) = 2.6116$
$\text{var}(\hat{q}_t) = 0.0037$	$\text{var}(\hat{q}_t) = 0.0062$	$\text{var}(\hat{q}_t) = 0.0004$

Panel 2 displays the variances of inflation and output gap associated for rules analyzed in the section 3.2 and 4.2: classical, PPI, CPI and Paraguayan Taylor rules. There are four different calibrations corresponding to alternative configurations for  $\Phi_{\pi}$  and  $\Phi_Y$ . PPI Taylor rule is the preferred policy rule for levels of  $\Phi_{\pi} = 1.5$  and  $\Phi_Y = 0.125$ , the calibration proposed by Taylor (1993) as a good approximation to the interest rate policy of the Fed during the Greenspan years<sup>16</sup>. When we consider the effect of no response to output fluctuations with a very aggressive anti-inflation stance in the case of the third rule ( $\Phi_Y = 5$ ), the Classic and PPI Taylor rules are preferred. Finally, the fourth rule assumes a strong output-stabilization motive ( $\Phi_Y = 1$ ), where the PPI Taylor regime is one more time the ideal rule. A result that shows up is that versions of the calibration that involve a response to output variations ( $\Phi_Y = 0.125$  and  $\Phi_Y = 1$ ) generate larger fluctuations in the output gap and inflation. Those variances are moderate under Taylor's original calibration, but they become bigger when the output coefficient  $\Phi_Y$  is set to unity. Taking into account the Taylor rule for Paraguay, the smallest variances values are attained when the monetary authority responds with an anti-inflation plan. Although the last calibration gives the smallest fluctuation in the output gap, it also gives the bigger fluctuation in the inflation. Hence, for the case of Paraguay and in the context of the New Keynesian model, a Taylor-type rule that responds aggressively to movements in inflation can approximate arbitrarily well the optimal policy.

**Table 2: Variances of control variables under alternative Taylor rules with different calibrations**

Policy Rule	Classic Taylor	PPI Taylor	CPI Taylor	Paraguay Taylor
$\Phi_{\pi} = 1.5$ $\Phi_Y = 0.125$	$\text{var}(\hat{y}_t) = 0.0055$ $\text{var}(\hat{\pi}_t) = 0.0099$	$\text{var}(\hat{y}_t) = 0.0058$ $\text{var}(\hat{\pi}_t) = 0.0076$	$\text{var}(\hat{y}_t) = 0.0060$ $\text{var}(\hat{\pi}_t) = 0.0135$	$\text{var}(\hat{y}_t) = 0.0222$ $\text{var}(\hat{\pi}_t) = 0.0033$
$\Phi_{\pi} = 1.5$ $\Phi_Y = 0$	$\text{var}(\hat{y}_t) = 0.0058$ $\text{var}(\hat{\pi}_t) = 0.0076$	$\text{var}(\hat{y}_t) = 0.0058$ $\text{var}(\hat{\pi}_t) = 0.0076$	$\text{var}(\hat{y}_t) = 0.0060$ $\text{var}(\hat{\pi}_t) = 0.0135$	$\text{var}(\hat{y}_t) = 0.0233$ $\text{var}(\hat{\pi}_t) = 0.0039$
$\Phi_{\pi} = 5$ $\Phi_Y = 0$	$\text{var}(\hat{y}_t) = 0.0052$ $\text{var}(\hat{\pi}_t) = 0.0003$	$\text{var}(\hat{y}_t) = 0.0052$ $\text{var}(\hat{\pi}_t) = 0.0003$	$\text{var}(\hat{y}_t) = 0.0074$ $\text{var}(\hat{\pi}_t) = 0.0011$	$\text{var}(\hat{y}_t) = 0.0187$ $\text{var}(\hat{\pi}_t) = 0.0005$
$\Phi_{\pi} = 1.5$ $\Phi_Y = 1$	$\text{var}(\hat{y}_t) = 0.0041$ $\text{var}(\hat{\pi}_t) = 0.0349$	$\text{var}(\hat{y}_t) = 0.0058$ $\text{var}(\hat{\pi}_t) = 0.0076$	$\text{var}(\hat{y}_t) = 0.0060$ $\text{var}(\hat{\pi}_t) = 0.0135$	$\text{var}(\hat{y}_t) = 0.0171$ $\text{var}(\hat{\pi}_t) = 0.0100$

Tables 3 shows the different fluctuations of output gap and inflation for different values of  $\lambda$  and  $\theta$ . The fluctuations of the two variables are lower when  $\lambda$  is high, so the country benefits when the economy is relatively open. Moreover, when the elasticity of substitution between domestic and foreign goods has

<sup>15</sup> The average welfare loss per period of a specific rule is given by the linear combination of the variances of the objective variables of that regime.

<sup>16</sup> Taylor's proposed coefficient values were 1.5 for inflation and 0.5 for output, based on a specification with *annualized* inflation and interest rates.

smaller values, the variances of output gap and inflation are also smaller. Although the above calibration is superior, the quantitative variances are not very significant.

**Table 3: Variances of control variables under the Taylor rule for Paraguay varying the degree of openness and intratemporal elasticity of substitution**

$\lambda/\theta$	1	2	3	4	5	6
1/2	$\text{var}(\hat{y}_t)=0.0047$ $\text{var}(\widehat{\Pi C}_t)=0.0039$	$\text{var}(\hat{y}_t)=0.0128$ $\text{var}(\widehat{\Pi C}_t)=0.0042$	$\text{var}(\hat{y}_t)=0.0259$ $\text{var}(\widehat{\Pi C}_t)=0.0044$	$\text{var}(\hat{y}_t)=0.0401$ $\text{var}(\widehat{\Pi C}_t)=0.0046$	$\text{var}(\hat{y}_t)=0.0538$ $\text{var}(\widehat{\Pi C}_t)=0.0047$	$\text{var}(\hat{y}_t)=0.0667$ $\text{var}(\widehat{\Pi C}_t)=0.0049$
1/3	$\text{var}(\hat{y}_t)=0.0047$ $\text{var}(\widehat{\Pi C}_t)=0.0020$	$\text{var}(\hat{y}_t)=0.0102$ $\text{var}(\widehat{\Pi C}_t)=0.0023$	$\text{var}(\hat{y}_t)=0.0193$ $\text{var}(\widehat{\Pi C}_t)=0.0026$	$\text{var}(\hat{y}_t)=0.0297$ $\text{var}(\widehat{\Pi C}_t)=0.0028$	$\text{var}(\hat{y}_t)=0.0402$ $\text{var}(\widehat{\Pi C}_t)=0.0031$	$\text{var}(\hat{y}_t)=0.0506$ $\text{var}(\widehat{\Pi C}_t)=0.0032$
1/4	$\text{var}(\hat{y}_t)=0.0048$ $\text{var}(\widehat{\Pi C}_t)=0.0014$	$\text{var}(\hat{y}_t)=0.0088$ $\text{var}(\widehat{\Pi C}_t)=0.0016$	$\text{var}(\hat{y}_t)=0.0155$ $\text{var}(\widehat{\Pi C}_t)=0.0018$	$\text{var}(\hat{y}_t)=0.0233$ $\text{var}(\widehat{\Pi C}_t)=0.0020$	$\text{var}(\hat{y}_t)=0.0316$ $\text{var}(\widehat{\Pi C}_t)=0.0022$	$\text{var}(\hat{y}_t)=0.0400$ $\text{var}(\widehat{\Pi C}_t)=0.0024$
1/5	<b><math>\text{var}(\hat{y}_t)=0.0048</math></b> <b><math>\text{var}(\widehat{\Pi C}_t)=0.0011</math></b>	$\text{var}(\hat{y}_t)=0.0080$ $\text{var}(\widehat{\Pi C}_t)=0.0012$	$\text{var}(\hat{y}_t)=0.0131$ $\text{var}(\widehat{\Pi C}_t)=0.0014$	$\text{var}(\hat{y}_t)=0.0220$ $\text{var}(\widehat{\Pi C}_t)=0.0019$	$\text{var}(\hat{y}_t)=0.0259$ $\text{var}(\widehat{\Pi C}_t)=0.0018$	$\text{var}(\hat{y}_t)=0.0328$ $\text{var}(\widehat{\Pi C}_t)=0.0019$

Table 4 shows the variances of output gap and inflation for different values of  $\rho$  and  $\theta$ . The optimal calibration can be found when the elasticity of substitution between domestic and foreign goods has a value of 3 and the coefficient of risk aversion has a value of 4. This calibration the implied outperforms other implied variances of the intertemporal and intratemporal elasticity of substitution.

**Table 4: Variances of control variables under the Taylor rule for Paraguay varying the intertemporal and intratemporal elasticity of substitution**

$\rho/\theta$	1	2	3	4	5	6
1	$\text{var}(\hat{y}_t)=0.0044$ $\text{var}(\widehat{\Pi C}_t)=0.0033$	$\text{var}(\hat{y}_t)=0.0128$ $\text{var}(\widehat{\Pi C}_t)=0.0037$	$\text{var}(\hat{y}_t)=0.0262$ $\text{var}(\widehat{\Pi C}_t)=0.0040$	$\text{var}(\hat{y}_t)=0.0409$ $\text{var}(\widehat{\Pi C}_t)=0.0043$	$\text{var}(\hat{y}_t)=0.0555$ $\text{var}(\widehat{\Pi C}_t)=0.0046$	$\text{var}(\hat{y}_t)=0.0694$ $\text{var}(\widehat{\Pi C}_t)=0.0048$
2	$\text{var}(\hat{y}_t)=0.0103$ $\text{var}(\widehat{\Pi C}_t)=0.0131$	$\text{var}(\hat{y}_t)=0.0536$ $\text{var}(\widehat{\Pi C}_t)=0.0146$	$\text{var}(\hat{y}_t)=0.1096$ $\text{var}(\widehat{\Pi C}_t)=0.0159$	$\text{var}(\hat{y}_t)=0.1675$ $\text{var}(\widehat{\Pi C}_t)=0.0170$	$\text{var}(\hat{y}_t)=0.2230$ $\text{var}(\widehat{\Pi C}_t)=0.0180$	$\text{var}(\hat{y}_t)=0.2749$ $\text{var}(\widehat{\Pi C}_t)=0.0188$
3	$\text{var}(\hat{y}_t)=0.0322$ $\text{var}(\widehat{\Pi C}_t)=0.0298$	$\text{var}(\hat{y}_t)=0.1400$ $\text{var}(\widehat{\Pi C}_t)=0.0333$	$\text{var}(\hat{y}_t)=0.2704$ $\text{var}(\widehat{\Pi C}_t)=0.0361$	$\text{var}(\hat{y}_t)=0.4018$ $\text{var}(\widehat{\Pi C}_t)=0.0386$	$\text{var}(\hat{y}_t)=0.5264$ $\text{var}(\widehat{\Pi C}_t)=0.0407$	$\text{var}(\hat{y}_t)=0.6417$ $\text{var}(\widehat{\Pi C}_t)=0.0425$
4	$\text{var}(\hat{y}_t)=0.0699$ $\text{var}(\widehat{\Pi C}_t)=0.0535$	$\text{var}(\hat{y}_t)=0.2717$ $\text{var}(\widehat{\Pi C}_t)=0.0596$	<b><math>\text{var}(\hat{y}_t)=0.0003</math></b> <b><math>\text{var}(\widehat{\Pi C}_t)=0.0001</math></b>	$\text{var}(\hat{y}_t)=0.7436$ $\text{var}(\widehat{\Pi C}_t)=0.0690$	$\text{var}(\hat{y}_t)=0.9653$ $\text{var}(\widehat{\Pi C}_t)=0.0727$	$\text{var}(\hat{y}_t)=1.1696$ $\text{var}(\widehat{\Pi C}_t)=0.0759$
5	$\text{var}(\hat{y}_t)=0.1234$ $\text{var}(\widehat{\Pi C}_t)=0.0841$	$\text{var}(\hat{y}_t)=0.4489$ $\text{var}(\widehat{\Pi C}_t)=0.0935$	$\text{var}(\hat{y}_t)=0.8234$ $\text{var}(\widehat{\Pi C}_t)=0.1014$	$\text{var}(\hat{y}_t)=1.1930$ $\text{var}(\widehat{\Pi C}_t)=0.1082$	$\text{var}(\hat{y}_t)=1.5397$ $\text{var}(\widehat{\Pi C}_t)=0.1140$	$\text{var}(\hat{y}_t)=1.8585$ $\text{var}(\widehat{\Pi C}_t)=0.1190$
6	$\text{var}(\hat{y}_t)=0.1928$ $\text{var}(\widehat{\Pi C}_t)=0.1215$	$\text{var}(\hat{y}_t)=0.6714$ $\text{var}(\widehat{\Pi C}_t)=0.1351$	$\text{var}(\hat{y}_t)=1.2155$ $\text{var}(\widehat{\Pi C}_t)=0.1465$	$\text{var}(\hat{y}_t)=1.7499$ $\text{var}(\widehat{\Pi C}_t)=0.1562$	$\text{var}(\hat{y}_t)=1.8585$ $\text{var}(\widehat{\Pi C}_t)=0.1190$	$\text{var}(\hat{y}_t)=2.7085$ $\text{var}(\widehat{\Pi C}_t)=0.1717$

## 6. Conclusion

This research work formalizes a small open economy model as a limiting case of the two-country general equilibrium framework and derives different monetary policy rules. The model developed in this work encompasses a small open economy with special targeting and Taylor rules. As a final point, a special monetary policy of Paraguay, in the form of Taylor rule, is compared with the above regimes.

When analyzing the case in which monetary policy objective is represented by a targeting rule, we consider three different alternatives. The first one is a combination of domestic inflation; output gap and real exchange rate. In this rule, the weights depend on the structural parameters of the model. We have also considered two special cases of targeting rules: inflation and exchange rate. Both policies are

specified according to a combination of domestic inflation; output gap and real exchange rate. In these two alternatives, the weights are specified according to the target they follow.

When the model is closed by specifying a Taylor-type monetary policy rule we use the households' intertemporal equation and a relation between PPI and CPI Inflation. First we consider a classical Taylor type rule where the interest rate responds to deviations of inflation and the output gap from their target levels and the inflation and output weights are non-negative coefficients determined by the central bank. We also study a PPI and CPI inflation Taylor rules where the central bank responds to domestic inflation (PPI) and CPI inflation, respectively. Finally, when the monetary policy rule of Paraguay is considered, the nominal interest rate is a function of its own lag, which reflects the idea of smoothing the path of the policy interest rate, the output gap and inflation gap.

Under the benchmark calibration of targeting rules, a productivity shock leads to an increase of home consumption and a depreciation of the exchange rate in the case of inflation and exchange rate targeting. When the economy experiences external disturbances there is an increase in production and the impact on inflation is much stronger under an exchange rate targeting. Under the analytical evaluation of Taylor-type rules, the productivity shock leads to a reduction in the interest rate, as it is needed in order to support the transitory expansion in consumption and output. There is also a fall in domestic and CPI inflation, but this reduction is more muted and followed by a hump-shape pattern in the CPI Taylor rule. When the economy experiences external shocks, the results do not change significantly comparing the three rules. The impact on CPI inflation is higher of a productivity shock under the Taylor Rule for Paraguay than the two Taylor rules (PPI and CPI) and the same shock depreciates the currency temporarily. A unit innovation in  $c_t^*$  generates an increase in CPI Inflation and the nominal interest rate has the same response as in the case of a productivity shock.

The sensitivity analysis exercise demonstrates that inflation targeting, when comparing the classical and exchange rate targeting, is the preferred policy for low levels of variances if the economy follow the benchmark calibration. PPI Taylor rule is the preferred policy rule for the calibration proposed by Taylor (1993) and when a strong output-stabilization motive rule is assumed. When a very aggressive anti-inflation policy is applied, the Classic and PPI Taylor rules are preferred. When applying the Taylor rule for Paraguay, the smallest variances values are attained when the monetary authority responds with an anti-inflation plan. Moreover, when the degree of openness is higher and the elasticity of substitution between domestic and foreign goods is calibrated with low values, the variances of output gap and inflation are smaller. Finally, the optimal calibration can be found when the elasticity of substitution between domestic and foreign goods has a value of 3 and the coefficient of risk aversion has a value of 4.

This paper demonstrated that a small open economy, completely integrated with the rest of the world, should be concerned about inflation variability. Therefore, the optimal policy in a small open economy, like Paraguay, is a rule with a very high anti-inflation stance. Furthermore, if the country increases the degree of openness and the coefficient of risk aversion, and encompasses these results with a relative low level of elasticity of substitution between domestic and foreign goods, the economy may improve welfare by getting smaller fluctuations of output gap and inflation.

The analysis of the monetary open economy developed in this paper raise a number of issues that can be improved. An interesting extension could be introducing a government spending shock and analyzing fiscal policy by allowing proportional taxation to be an endogenous variable. Moreover, a decision can be made regarding the nature of international asset markets: the assumption of perfect capital markets can be relaxed in order to get a more realistic framework. Other interesting avenue for future research may include a complete analysis of total welfare losses for the different policies. Furthermore, a model can be estimated with real data of the Paraguayan economy and compare these results with the calibrate version of the model. Finally, when considering exchange rate targeting rule in the specific case of Paraguay, a decision can be made in targeting the exchange rate with respect to the United States, which matters for currency demand in the long run, and the exchange rate with respect to Brazil, which matters for cost push inflation in the short run.

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## APPENDIX A: A First Order Approximation of the equilibrium conditions

In this appendix, we derive the first order approximation to the equilibrium conditions of the model under the assumptions that  $\bar{c}_t = \bar{c}_t^*$ . To simplify we use the following isoelastic functional forms for utility from consumption and disutility from production

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho} \text{ and } V(y_t, \varepsilon_{Y,t}) = \frac{\varepsilon_{Y,t}^\eta y_t^{1+\eta}}{1+\eta}.$$

### A.1 Home Supply

The first order approximation to the price setting equation follow Benigno and Benigno (2001) and Benigno and Benigno (2003). This condition is derived from the first order condition of sellers that can reset their prices:

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left( \frac{\tilde{p}_j(j)}{P_{H,t}} \right)^{-\sigma} Y_{H,T} \left[ \frac{\tilde{p}_j(j) P_{H,t}}{P_{H,t} P_T} - \frac{\sigma}{(1-\tau_T)(\sigma-1)} \frac{V_y(\tilde{y}_{T,T}(j), \varepsilon_{Y,T})}{U_c(C_T)} \right] \right\} = 0$$

where

$$\tilde{y}_t(h) = \left( \frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_t$$

and

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)(\tilde{p}_t(h))^{1-\sigma}$$

To derive the first order approximation of the Real Exchange Rate we take into account that in the rest of the world  $P_F = SP^*$ , so this variable can be expressed as:

$$\frac{P_t}{P_{H,t}} = (1-\theta) + \left( q_t \frac{P_t}{P_{H,t}} \right)^{1-\theta}$$

And we get:

$$\tilde{p}_{H,t} = -\frac{\lambda \hat{q}_t}{(1-\lambda)}$$

After replacing the real exchange rate with the domestic relative prices in the price setting equation, we get the first order approximation of the Phillips curve:

$$\hat{\Pi}_t = k(\rho \hat{c}_t + \eta \hat{y}_t + \lambda(1-\lambda)^{-1} \hat{q}_t + \hat{\mu}_t - \eta \hat{\varepsilon}_t) + \beta E_t \hat{\Pi}_{t+1}$$

where  $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$

### A.2 Home Demand

As shown in the text, home demand equation is:

$$y^d(h) = \left[ \frac{p(h)}{P_H} \right]^{-\sigma} \left\{ \left[ \frac{P_H}{P} \right]^{-\theta} \left[ (1-\lambda)C + \lambda \left( \frac{1}{Q} \right)^{-\theta} C^* \right] \right\}$$

The first order approximation to demand in the small open economy is therefore:

$$\hat{y}_t = (1-\lambda)\hat{c}_t + \lambda\hat{c}_t^* + Y\hat{q}_t$$

where  $Y = \frac{\theta\lambda(2-\lambda)}{1-\lambda}$ .

### A.3 Risk Sharing Equation

In a perfectly integrated capital market, the value of the intertemporal marginal rate of substitution is equated across borders:

$$\frac{U_c(C_{t+1}^*)}{U_c(C_t^*)} \frac{P_t^*}{P_{t+1}^*} = \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{S_{t+1}P_t}{S_tP_{t+1}}$$

Assuming the symmetric steady state equilibrium, the log linear approximation to the above condition is:

$$\hat{c}_t = \hat{c}_t^* + (1/\rho) \hat{q}_t$$

## APPENDIX B: Expressions for weights of inflation, output and exchange rate.

De Paoli (2007) derived a second order approximation to the utility function following Benigno and Benigno (2003) in order to get the small open economy loss function as a quadratic terms of  $\hat{y}_t$ ,  $\hat{q}_t$  and  $\hat{\Pi}_t$ . From this derivation we get the weights of inflation, output and exchange rate as functions of the structural parameters of the model.

B.1 Weight of Output

$$\Phi_y = (\eta + \rho)(1 - \Phi) + \frac{(\rho-1)(-l(1-\Phi)-(\lambda-\Phi))}{(1+l)} + Lx_1 \left[ (\eta + \rho) + \eta(\eta + 1) - \frac{\rho(\rho-1)}{(1+l)} \right] - \frac{Lx_2(1-\lambda)^2\lambda(\rho\theta-1)}{(1+l)}$$

B.2 Weight of Exchange Rate

$$\Phi_y = -\frac{(\lambda + l)(\rho - 1)}{(1 - \lambda)\rho^2} + \frac{Lx_1 l(\rho - 1 - l)}{(1 - \lambda)^2 \rho} + \frac{Lx_2 \lambda(\rho\theta - 1)[\rho\theta(1 - \lambda) + \lambda + l]}{\rho^2} + \frac{Lx_2 \lambda(\theta - 1)}{1 - \lambda}$$

B.3 Weight of Inflation

$$\Phi_{\pi} = \frac{\sigma}{\mu\kappa} + (1 + \eta)\frac{\sigma}{\kappa}Lx_1$$

where

$$l = (-1)(2 -)$$

$$u = \frac{1}{(1 - \lambda)}$$

$$(1 - \Phi) = \frac{1}{u}$$

$$0 \leq \Phi < 1; u > 1$$

$$Lx_1 = \frac{1}{(\rho + \eta) + ln} [lu^{-1} + (1 - \lambda) - u^{-1}]$$

$$Lx_2 = \frac{1}{(\rho + \eta) + ln} [\rho(u^{-1} - (1 - \lambda)) + (1 - \lambda)(n + \rho)]$$

$$Lx_3 = \frac{1}{(\rho + \eta) + ln} [(\rho\theta - 1)(1 - \lambda)u^{-1} - (n\theta + 1)]$$

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<sup>i</sup> The government spending ( $G_t$ ) had been removed from the original model in order to simplify the equation.

